



The diagram shows two concentric circles representing Fermi surfaces in k-space. The outer circle has four gray dots representing occupied states. Dashed lines connect the center to these dots. Labels include $k \uparrow$ at the top right, $-k \uparrow + q$ at the top left, and $-k' + q \downarrow$ at the bottom right. Dotted arrows point from the center towards the bottom-left and bottom-right dots.

Fulde-Ferrell-Larkin-Ovchinnikov State in Heavy Fermion CeCoIn_5

$$\Delta(\mathbf{r}) = \Delta_0 e^{i\mathbf{q} \cdot \mathbf{r}}$$

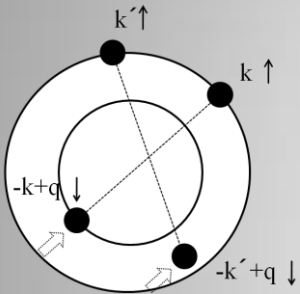
Introduction

FFLO state : Unconventional state(SC) predicted by Fulde, Ferrell, Ovchinnikov, and Larkin in 1964

Cooper pair with finite center-of-mass momentum q in high field H

Pairing in FFLO state

$(k\uparrow, -k+q\downarrow)$



$$\Delta(\mathbf{r}) = \Delta_0 e^{iq \cdot \mathbf{r}}$$

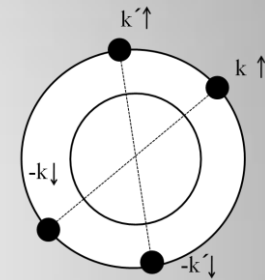
Order parameter : **oscillatory**

$$q = \frac{2\mu_B H}{\hbar v_F}$$

- More realistic state than BCS state under a certain condition
- Order parameters of the two degenerate states
 - $\Delta(\mathbf{r}) = \Delta_0 e^{iq \cdot \mathbf{r}}$: FF state
 - $\Delta(\mathbf{r}) = \Delta_0 e^{-iq \cdot \mathbf{r}}$: FF state
- linear combination
 - $\Delta(\mathbf{r}) = 2\Delta_0 \cos(q \cdot \mathbf{r})$: LO state

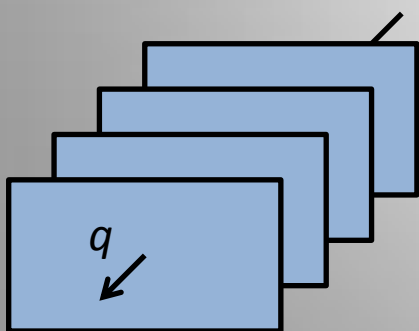
Pairing in BCS state

$(k\uparrow, -k\downarrow)$



$$\Delta(\mathbf{r}) = \Delta_0$$

Order parameter : **constant**



$$\Delta(\mathbf{r}) = 0$$

LO state:
Planar nodes
perpendicular to
wave vector q

The condition in which the FFLO state realizes

- orbital limit > Pauli limit
- clean limit
- 2-dimensional Fermi surface

Introduction

Orbital limit and Pauli limit in type II superconductor

• orbital limit

Linearized GL eq. $-\frac{\hbar^2}{2m^*} [\nabla - \frac{ie^*}{\hbar} A]^2 \Psi(r) + \alpha \Psi(r) + \beta \Psi^3(r) = 0$ (near the transition point)

$$\rightarrow H_{c2}^{\text{orb}}(T) = \frac{\phi_0}{2\pi\xi^2(T)} : \text{Condensed state is destroyed by vortexes.}$$

• Pauli limit

$$E_c = \frac{1}{2} N(0)\Delta^2 : \text{condensation energy}$$

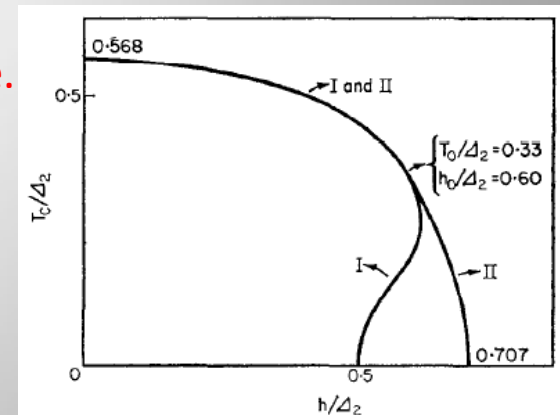
$$E_p = \frac{1}{2} \chi_n H^2 : \text{Zeeman energy } \left(\chi_n = \frac{1}{2} (g\mu_B)^2 N(0) \right)$$

$$E_c = E_p \rightarrow H_{c2}^P = \frac{\sqrt{2}\Delta}{g\mu_B} : \text{Condensed state is destroyed by Pauli paramagnetic effect.}$$

Phase transition can be first order below a certain temperature.

✳ Maki parameter

$$\alpha = \frac{\sqrt{2}H_{c2}^{\text{orb}}}{H_{c2}^P} \propto \frac{\Delta}{\varepsilon_F} \quad (\ll 1 \text{ conventional superconductor})$$



G. Sarma, J. Phys. Chem. Solids. **24**, 1029 (1963).

Introduction

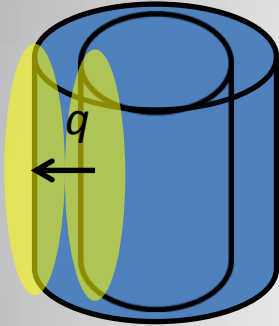
Clean limit

Nonmagnetic impurity is very harmful to the FFLO state.

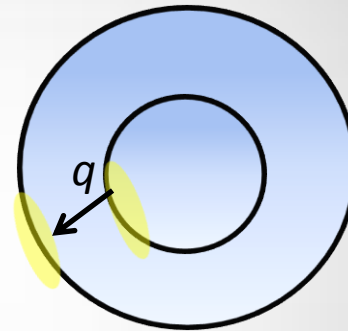
2-dimentional Fermi surface

Fermi surfaces split by Zeeman effect should be fit by translation.

→2-D more favorable than 3-D



2-dimentional Fermi surface

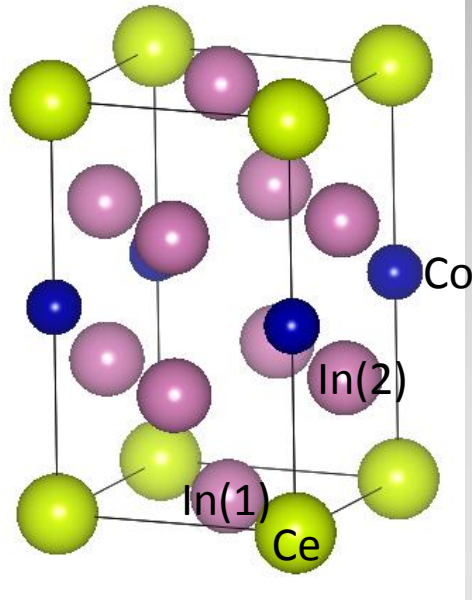


3-dimentional Fermi surface

Candidate systems for the FFLO state which meet the upper conditions

- organic SC
- cold atoms
- astrophysics
- nuclear physics

Heavy fermion CeCoIn₅

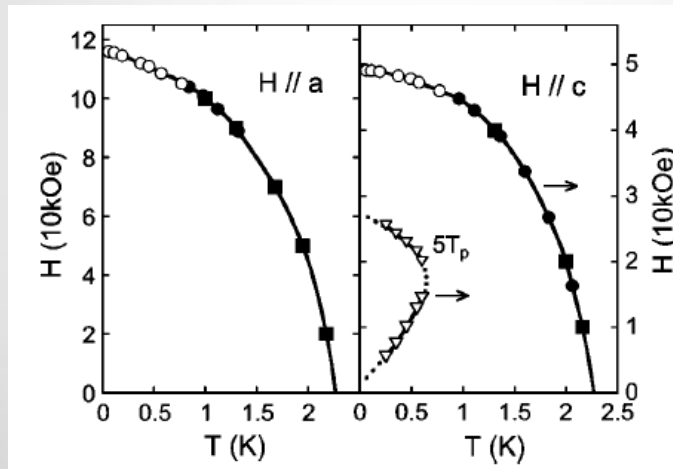


CeCoIn₅

Tetragonal structure $T_c = 2.3$ K

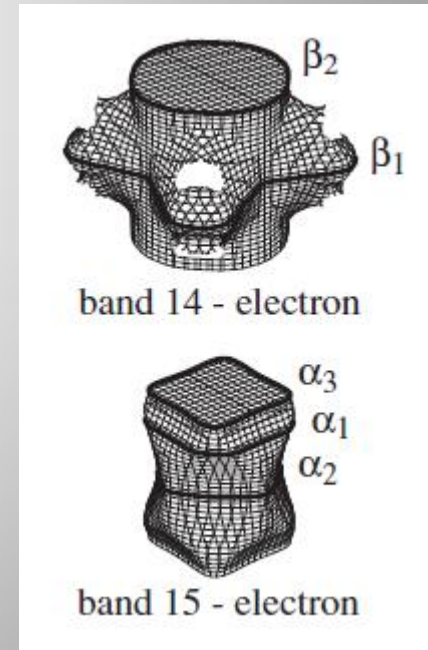
$\gamma \sim 1000$ mJ/K²mol ($\sim 1-10$ for normal metal)

- first order phase transition at $H_{c2} \rightarrow$ indicating Pauli limit
- CeIn₃-CoIn₂ layered-structure \rightarrow 2-dimensional



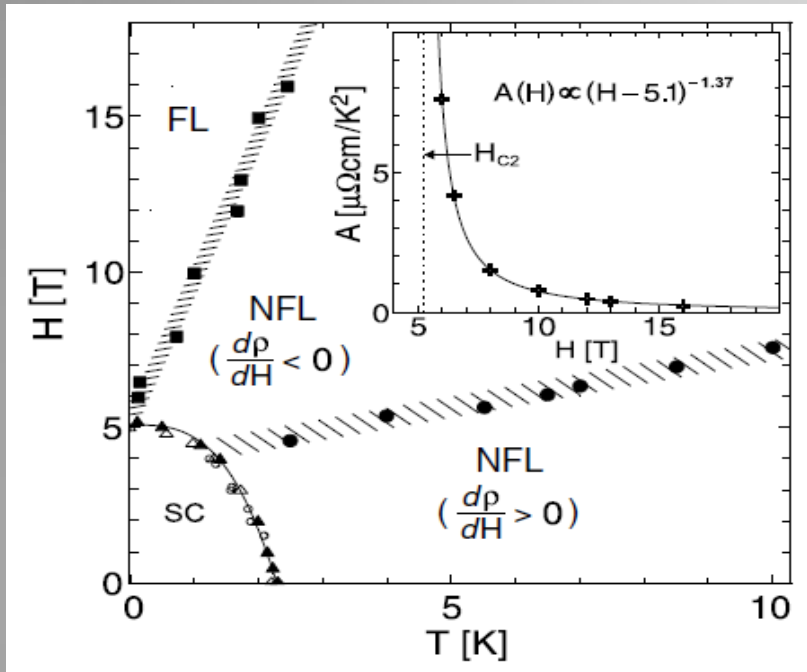
T. Tayama *et al.*, Phys. Rev. B. **65**, 180504 (2002).

- ✓ Maki parameter : $\alpha = 4.6(H // a), 5.0(H // c)$
- ✓ mean free path : $l \sim 3000$ nm (~ 10 nm for free electron)
- ✓ 2-dimensional Fermi surface \rightarrow

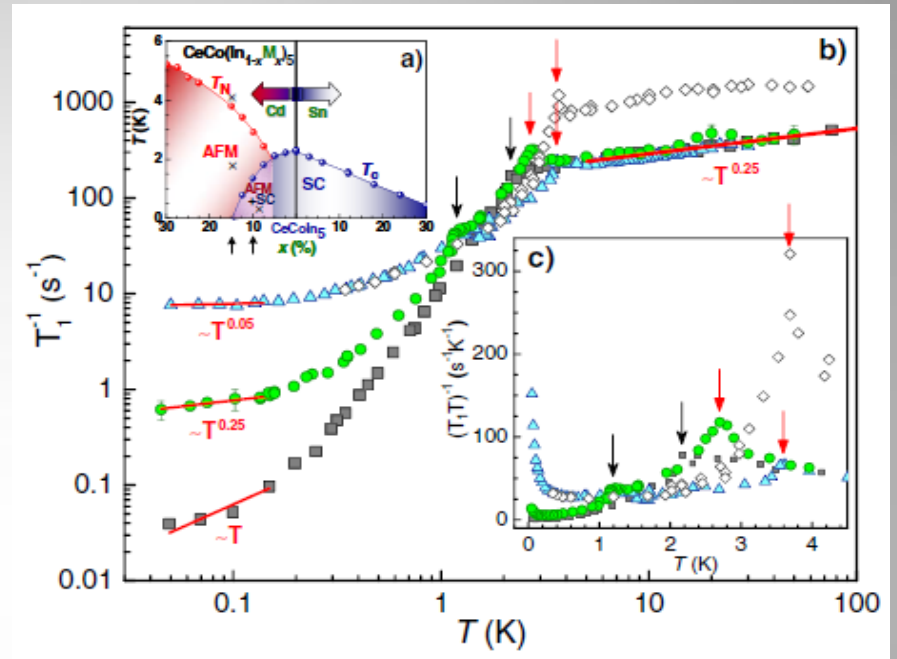


H. Shishido *et al.*, J. Phys. Soc. Jpn. **74**, 1103 (2005).

Heavy fermion CeCoIn₅



J. Paglione *et al.*, Phys. Rev. Lett. **91**, 246405 (2003).



R. R. Urbano *et al.*, Phys. Rev. Lett. **99**, 146402 (2007).

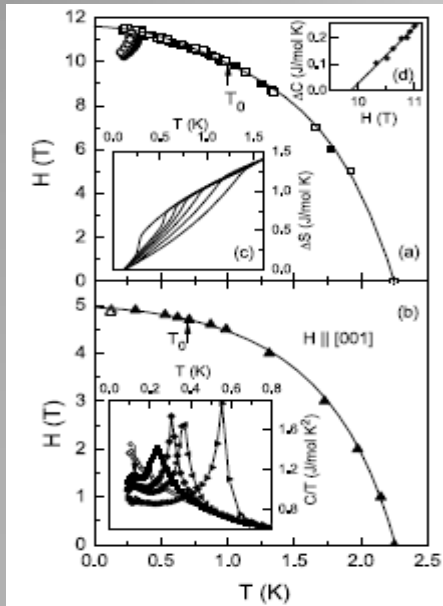
- AFM induced by Cd dope(hole)

- $\rho - \rho_0 \propto T$: Non-FL
- Divergence of A value near the $H_{c2}(T = 0)$
- Quantum critical point near the $H_{c2}(T = 0)$



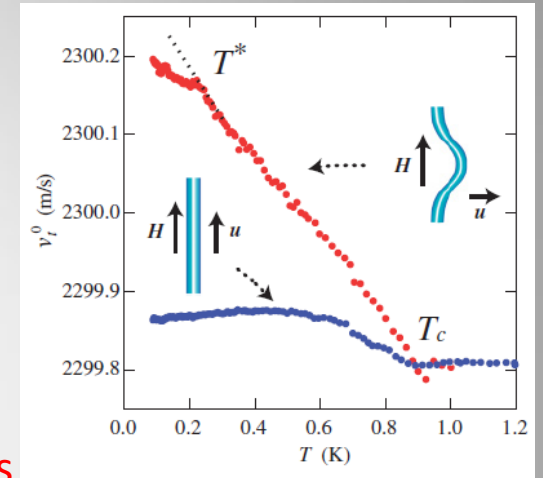
Heavy fermion CeCoIn₅

New phase in high-field and low-temperature(HFLT) region



- Specific heat measurement :
branch of the 2-order
phase transition line

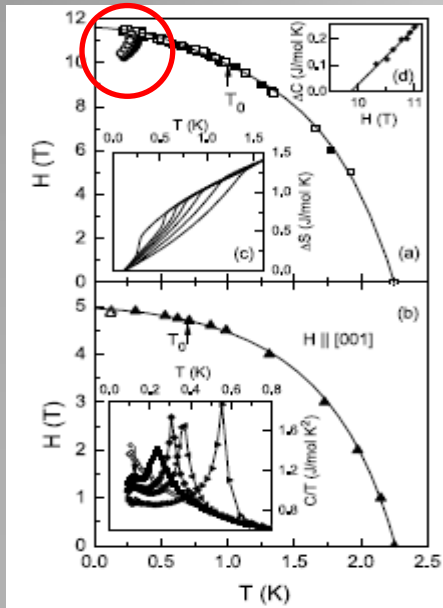
- Ultrasound measurement :
Lorentz mode (red line)
with a cusp at T^*
→ 'softening' of vortices
because of planar nodes



A. Bianchi *et al.*, Phys. Rev. Lett. **91**, 187004 (2003).

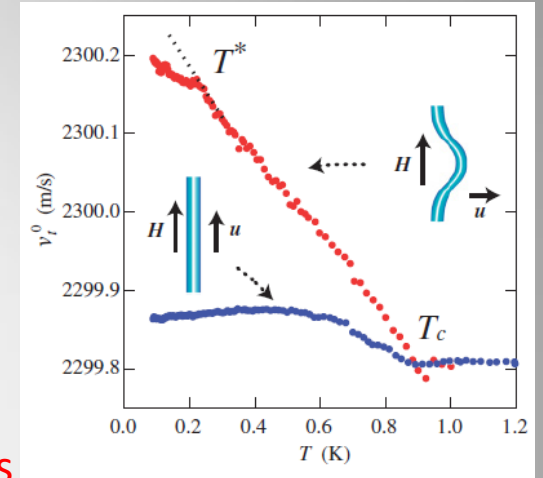
Y. Matsuda *et al.*, J. Phys. Soc. Jpn. **76**, 051005 (2007).

Heavy fermion CeCoIn₅



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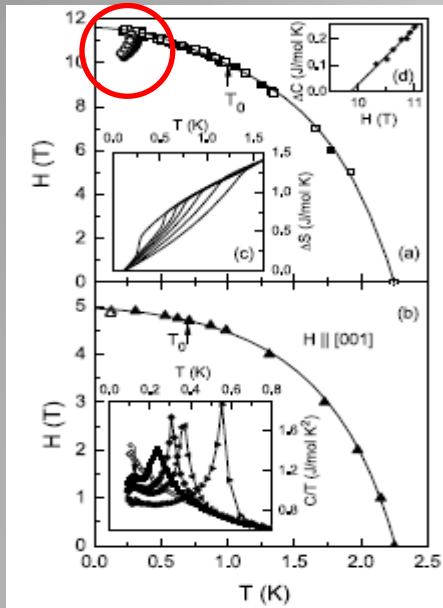
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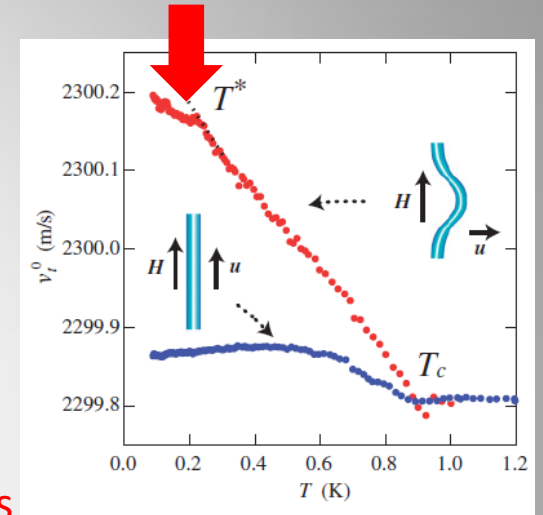
Heavy fermion CeCoIn₅



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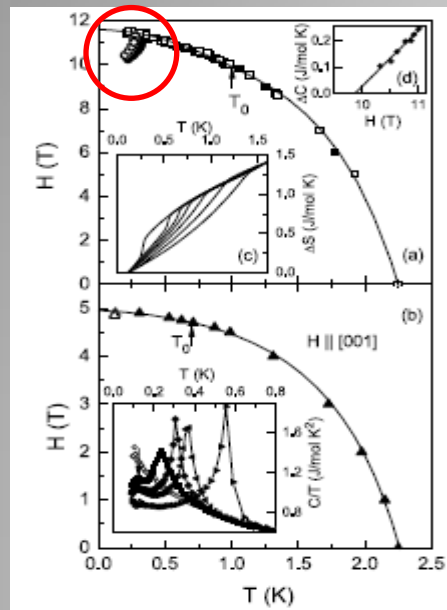
- Ultrasound measurement :
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→ 'softening' of vortices
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A. Bianchi *et al.*, Phys. Rev. Lett. **91**, 187004 (2003).

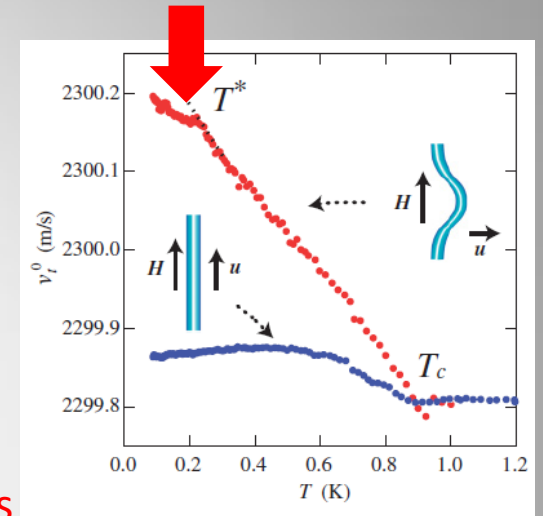
Y. Matsuda *et al.*, J. Phys. Soc. Jpn. **76**, 051005 (2007).

Heavy fermion CeCoIn_5



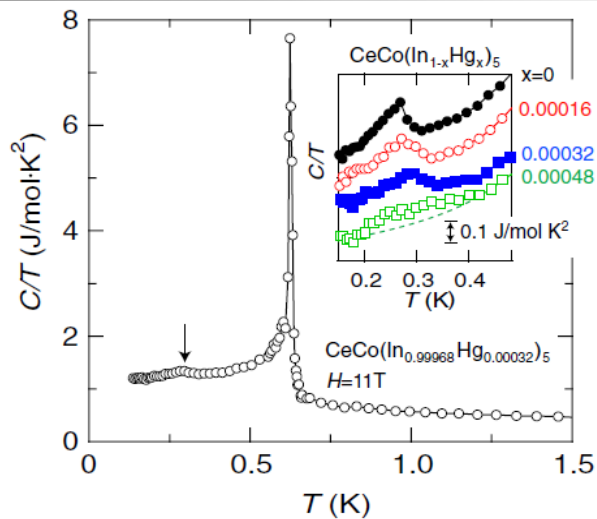
New phase in high-field and low-temperature (HFLT) region

- Specific heat measurement :
branch of the 2-order phase transition line
- Ultrasound measurement :
Lorentz mode (red line) with a cusp at T^*
→ 'softening' of vortices
because of planar nodes



A. Bianchi *et al.*, Phys. Rev. Lett. **91**, 187004 (2003).

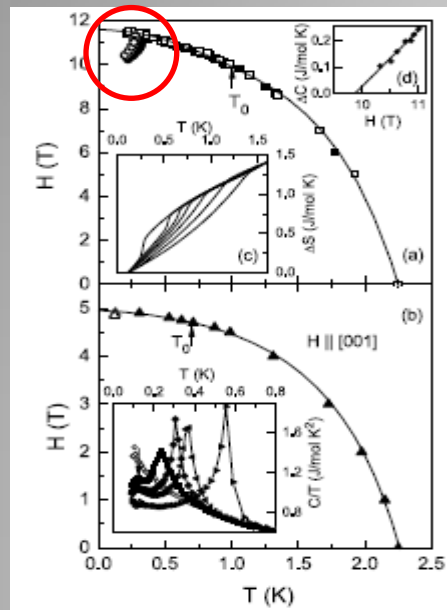
Y. Matsuda *et al.*, J. Phys. Soc. Jpn. **76**, 051005 (2007).



- Sensitive to impurity
vanishing with 0.05% Hg dope
→ consistent with FFLO

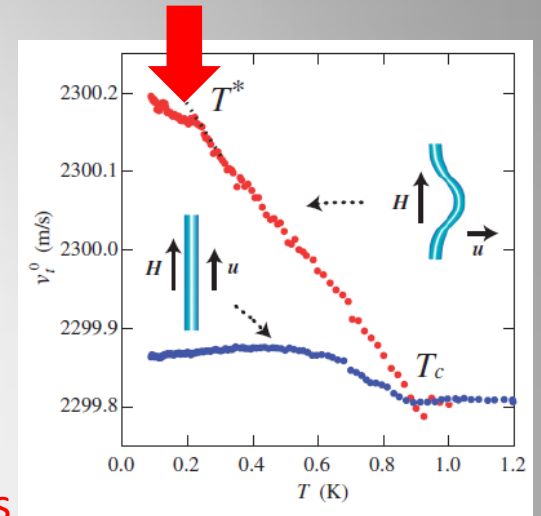
Y. Tokiwa *et al.*, Phys. Rev. Lett. **101**, 037001 (2008).

Heavy fermion CeCoIn₅



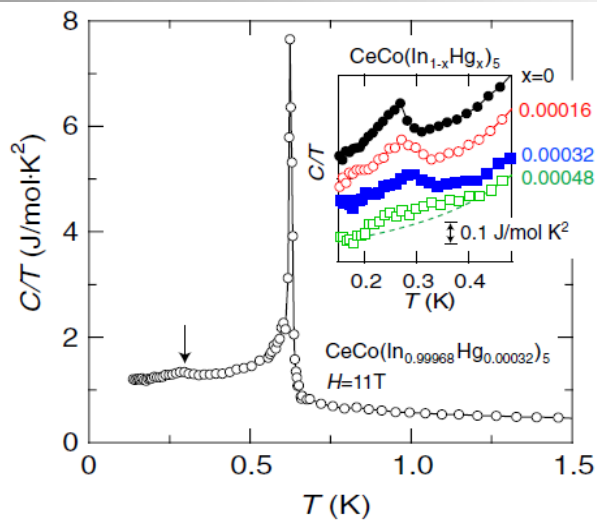
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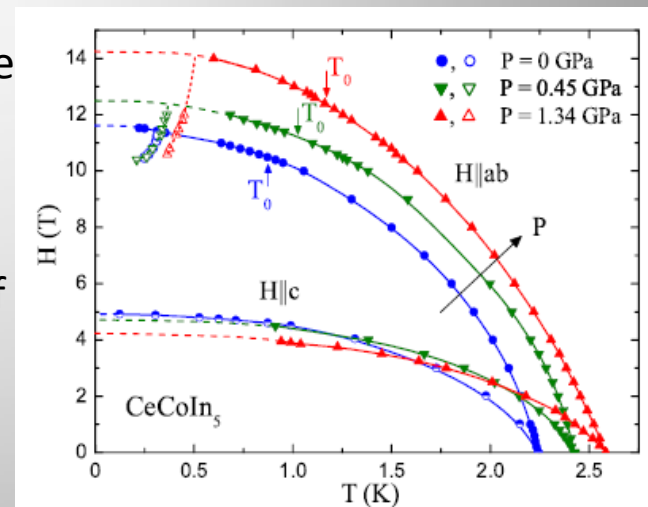


A. Bianchi *et al.*, Phys. Rev. Lett. **91**, 187004 (2003).

Y. Matsuda *et al.*, J. Phys. Soc. Jpn. **76**, 051005 (2007).



- Sensitive to impurity
vanishing with 0.05% Hg dope
→ consistent with FFLO
- stability against suppression of
AFM fluctuation by pressure
→ irrelevant to AFM



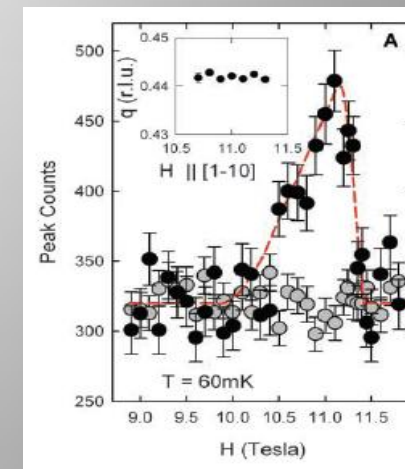
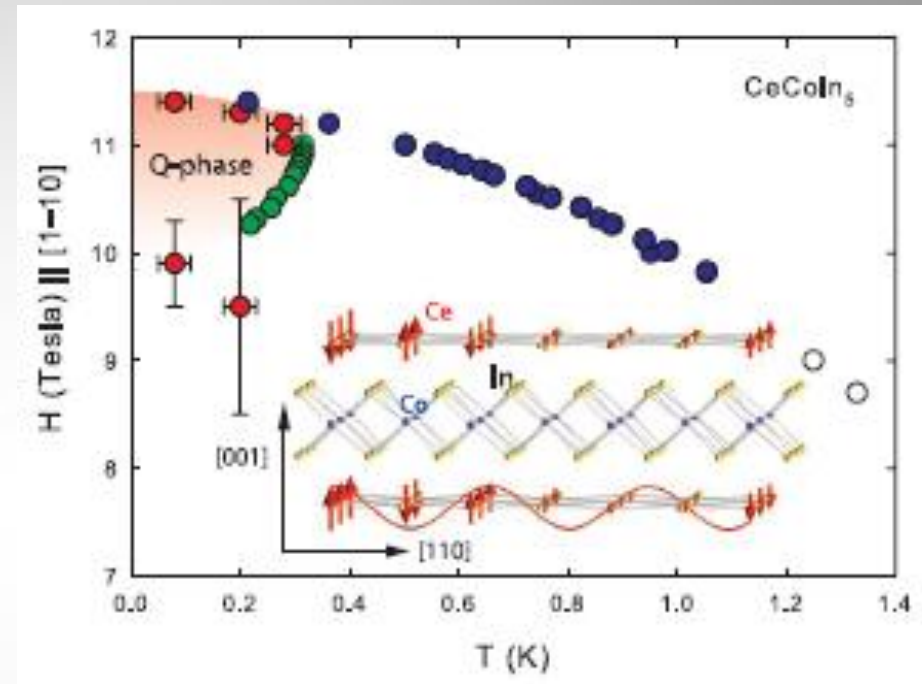
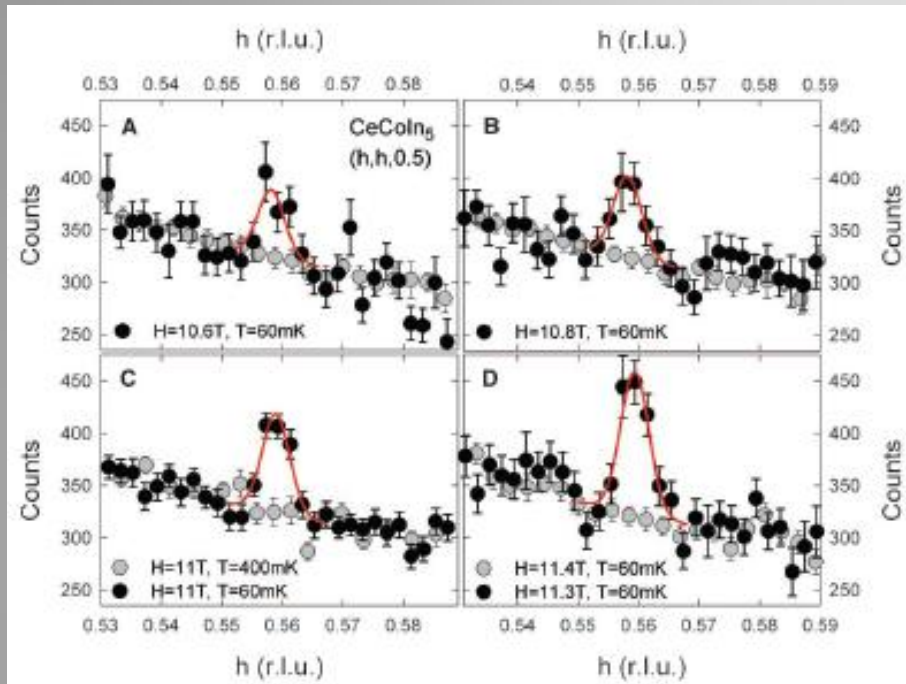
Y. Tokiwa *et al.*, Phys. Rev. Lett. **101**, 037001 (2008).

C. F. Miclea *et al.*, Phys. Rev. Lett. **96**, 117001 (2006).

Neutron diffraction measurement

M. Kenzelmann *et al.*, Science 321, 1652 (2008).

$H // [1-10] (h,h,0.5)$



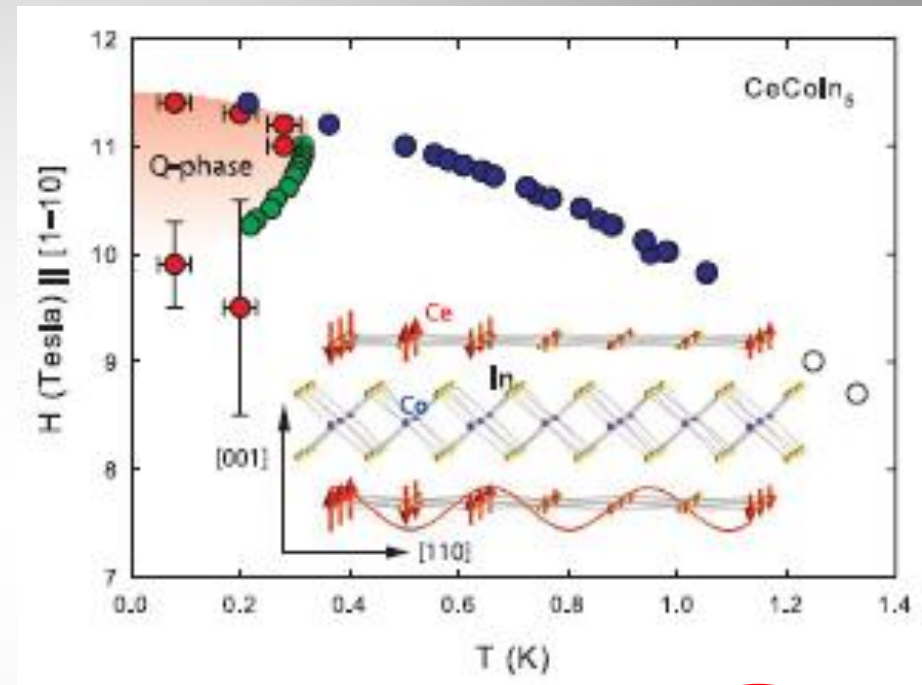
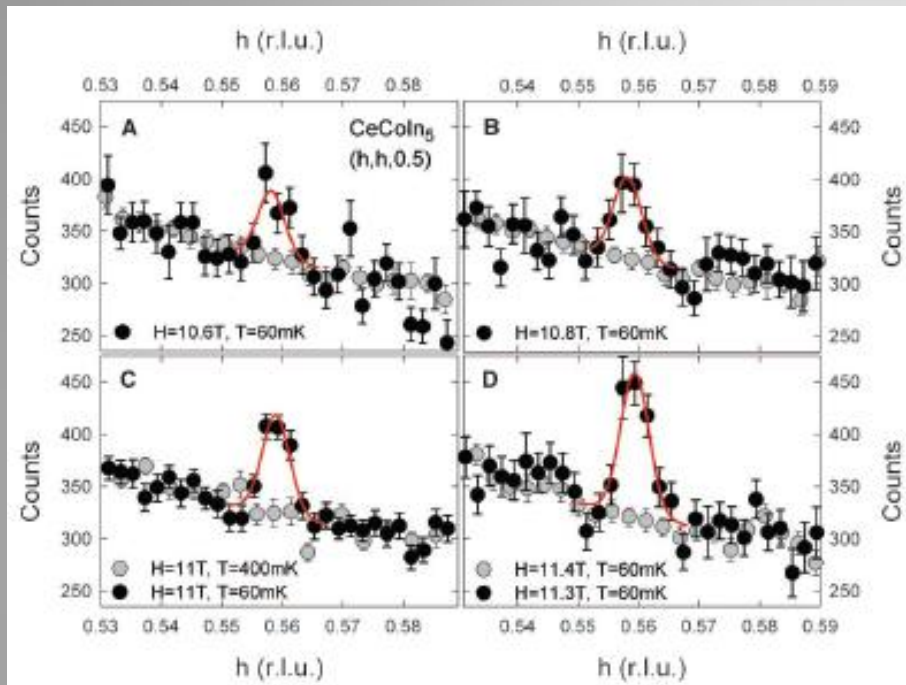
- magnetic order scale: 60 nm \gg vortex core scale: ~ 10 nm
- magnetic structure $\mathbf{Q} = \left(\frac{1}{2} + q, \frac{1}{2} + q, \frac{1}{2}\right) q = 0.06$ $\mathbf{S} \perp \mathbf{H} \perp \mathbf{q}$ $\mu_0 = 0.15 \mu_B$
- q : H -independent In FFLO state $q = \frac{2\mu_B H}{h v_F}$ \rightarrow inconsistent with FFLO

Incommensurate(IC) magnetic order
 \rightarrow SC gap function that carries finite momentum

Neutron diffraction measurement

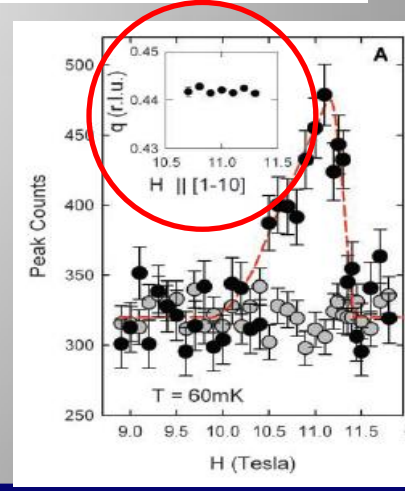
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$H // [1-10] (h,h,0.5)$

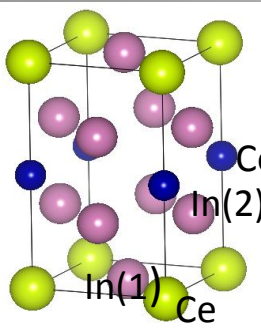


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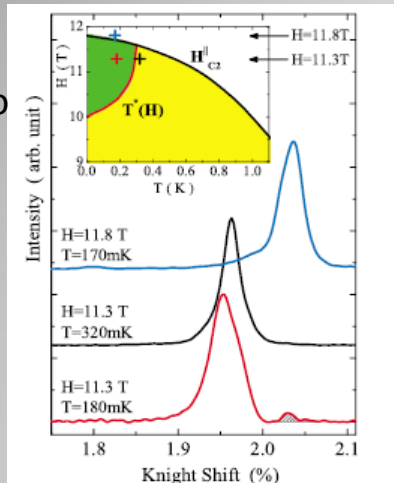
Incommensurate(IC) magnetic order
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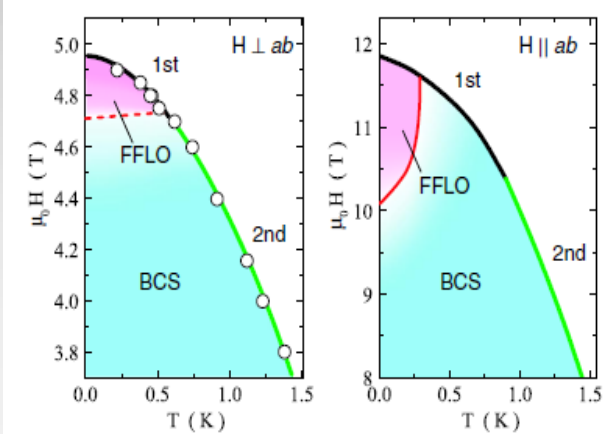
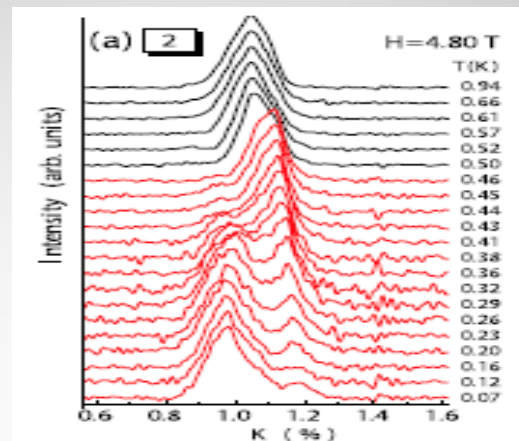
NMR measurement



In(1) site $H // [100]$



In(1) site $H // [001]$



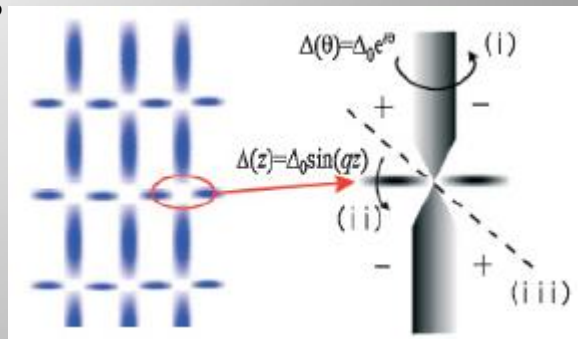
K. Kakuyanagi *et al.*, Phys. Rev. Lett. **94**, 047602 (2005).

K. Kumagai *et al.*, Phys. Rev. Lett. **97**, 227002 (2006).

- double peaks in HFLT region → coexistence of SC and normal state inhomogeneous FFLO state

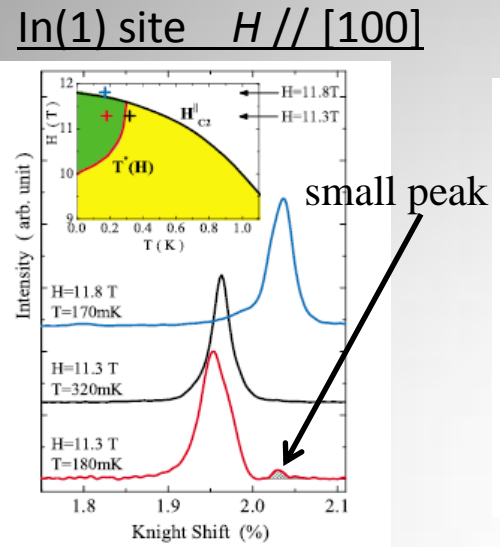
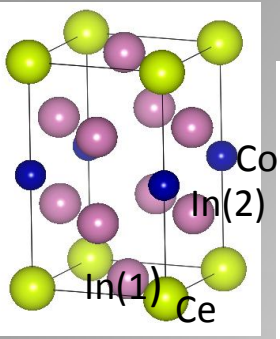
- well-separated peaks No quasiparticle excitation at the point where vortex intersects with node The sign of order parameter is same across the dot line in the picture.

→ No Andreev bound state

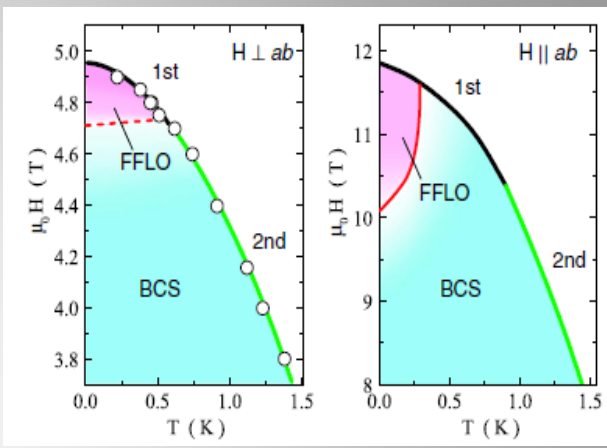
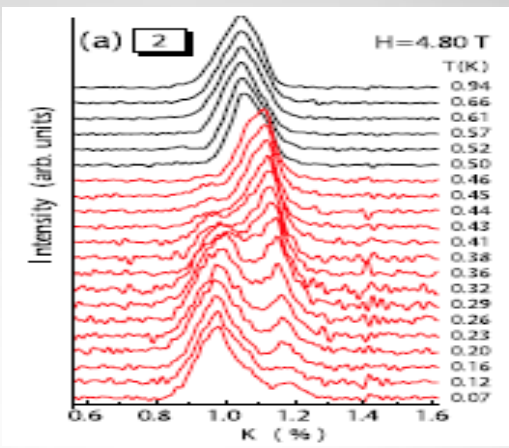


Y. Matsuda *et al.*, J. Phys. Soc. Jpn. **76**, 051005 (2007).

NMR measurement



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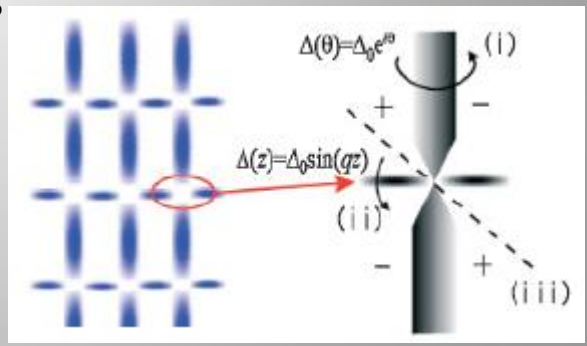
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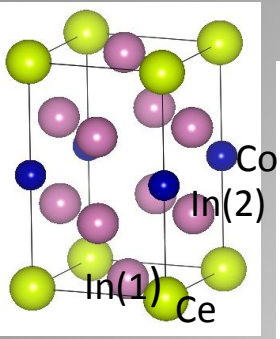
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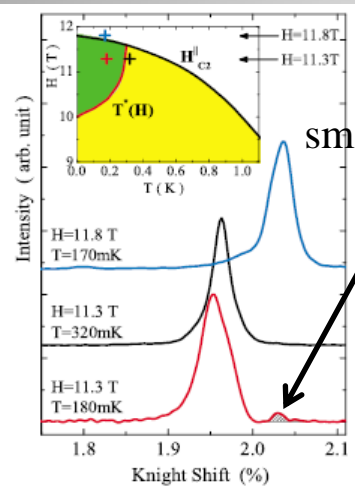


Y. Matsuda *et al.*, J. Phys. Soc. Jpn. **76**, 051005 (2007).

NMR measurement

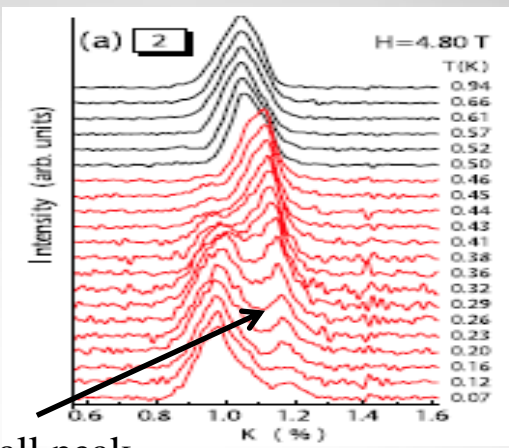


In(1) site $H // [100]$

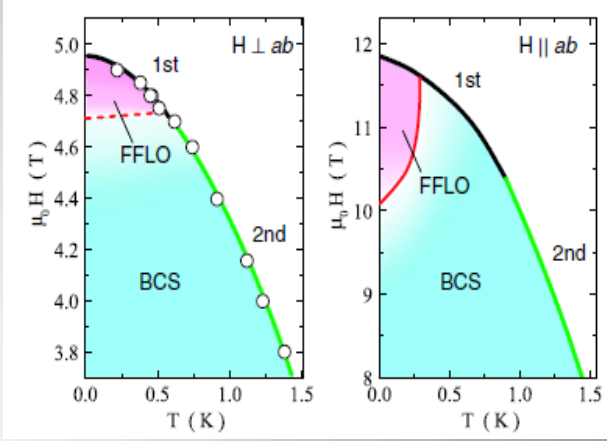


small peak

In(1) site $H // [001]$



small peak



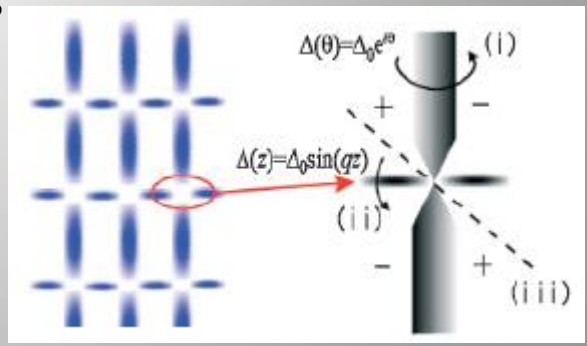
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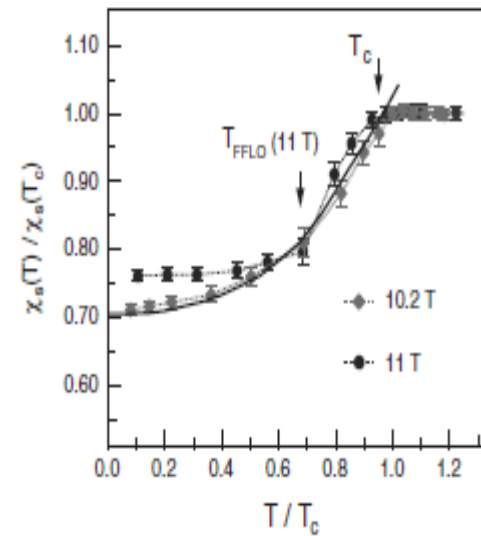
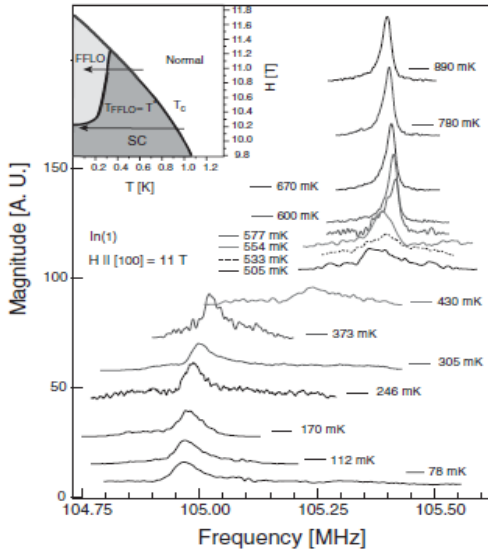
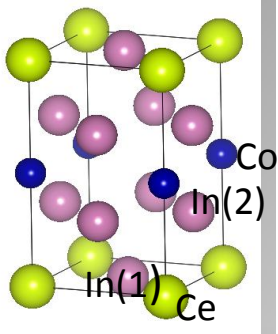
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Y. Matsuda *et al.*, J. Phys. Soc. Jpn. **76**, 051005 (2007).

NMR measurement



In(1) site $H // [100]$

In HFLT region

- no other peaks

- χ enhanced

T-independent

← DOS at E_F in FFLO

$\chi \propto \text{DOS}(E_F)$

V. F. Mitrovic *et al.*, Phys. Rev. Lett. **97**, 117002 (2006).

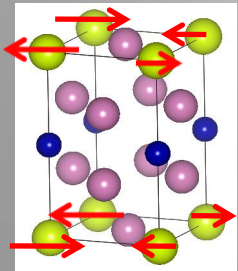
In(1), In(2), Co site $H // [100]$

- In(2) site : double broadening peaks

- Broadening depends on sites.

→ Not from FFLO or vortexes but from magnetic order in the vortex core

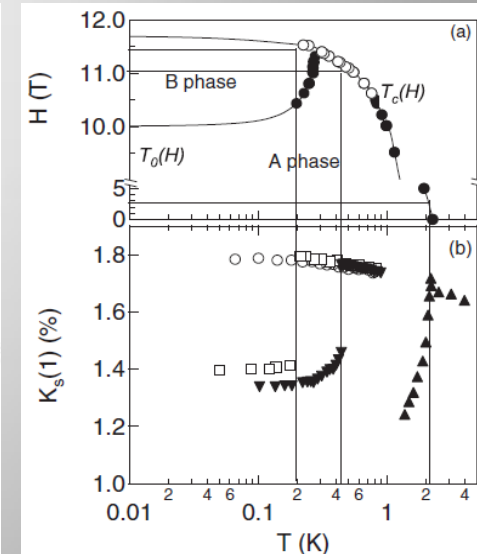
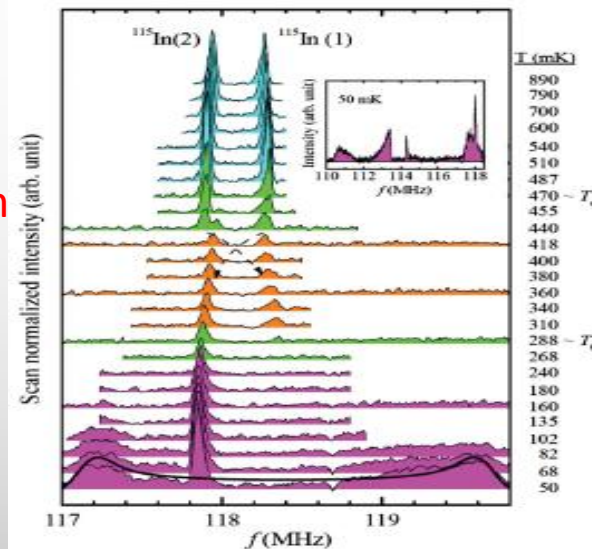
magnetic structure consistent with data



$$\mathbf{Q} = \left(\frac{1}{2} + q, \frac{1}{2}, \frac{1}{2} \right)$$

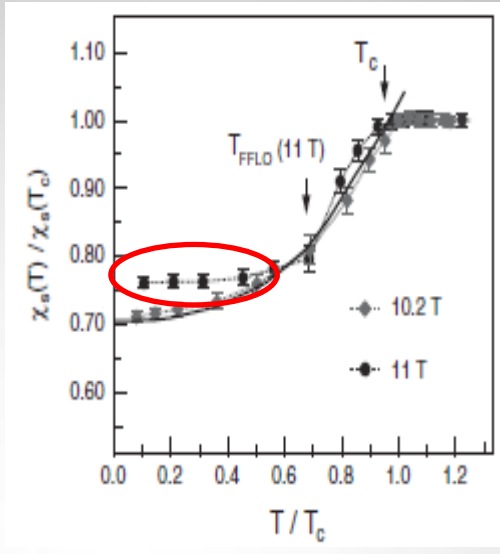
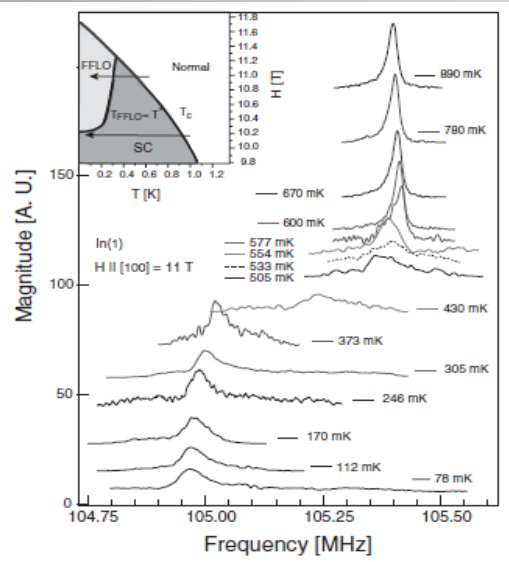
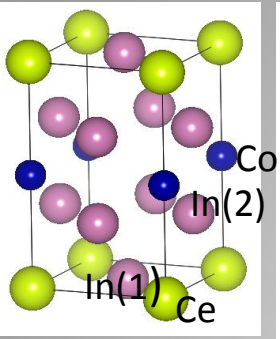
q : undetermined

spin : $\mathbf{S} // \mathbf{H} // [100]$



B. L. Young *et al.*, Phys. Rev. Lett. **98**, 036402 (2007).

NMR measurement

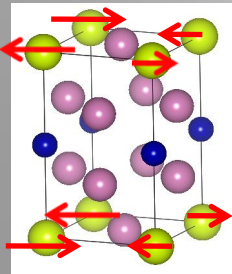


In(1) site $H // [100]$
 In HFLT region
 • no other peaks
 • χ enhanced
 T-independent
 ← DOS at E_F in FFLO
 $\chi \propto \text{DOS}(E_F)$

V. F. Mitrovic *et al.*, Phys. Rev. Lett. **97**, 117002 (2006).

In(1), In(2), Co site $H // [100]$

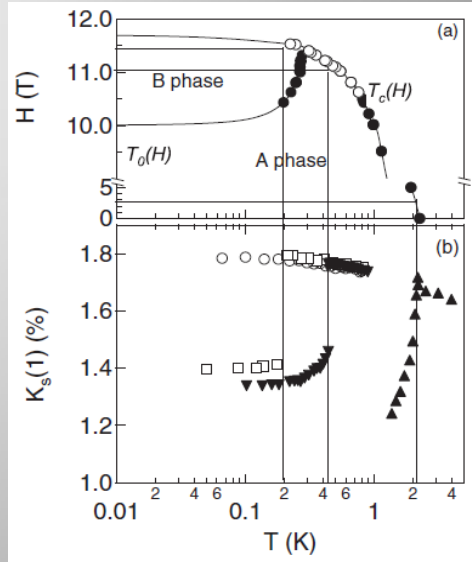
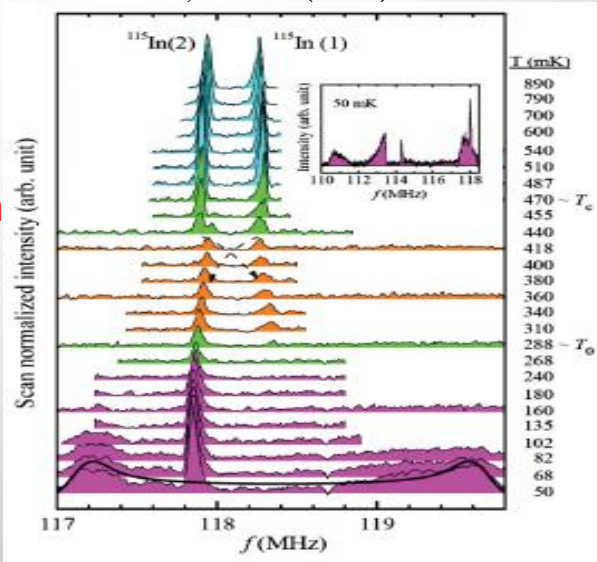
- In(2) site : double broadening peaks
 - Broadening depends on sites.
 - Not from FFLO or vortexes but from magnetic order in the vortex core
- magnetic structure consistent with data



$$\mathbf{Q} = \left(\frac{1}{2} + q, \frac{1}{2}, \frac{1}{2} \right)$$

q : undetermined

spin : $\mathbf{S} // \mathbf{H} // [100]$



B. L. Young *et al.*, Phys. Rev. Lett. **98**, 036402 (2007).

Magnetic Structure

N. J. Curro *et al.*, J. Low. Temp. Phys. **158**, 635 (2010).

NMR

$\mathbf{S} \perp \mathbf{H} \perp \mathbf{q}$ (neutron diffraction measurement) : dipolar hyperfine coupling

$\mathbf{S} // \mathbf{H} // [100]$ (NMR measurement) : isotropic hyperfine coupling

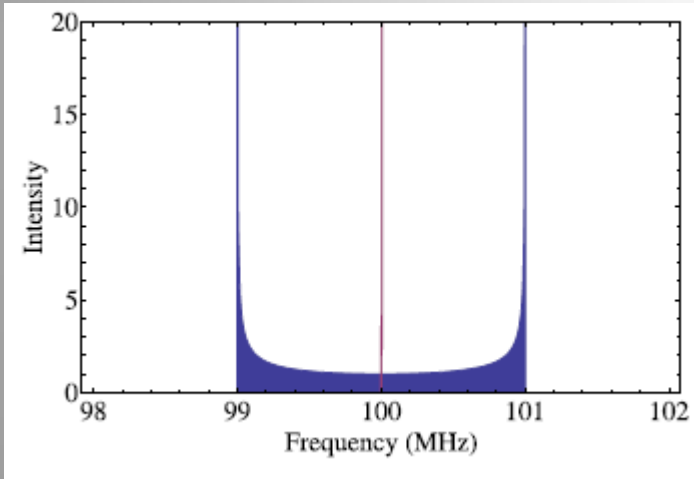
f_Q : quadrupolar interaction

γ : gyromagnetic ratio

Resonance frequency: $f = \gamma | \mathbf{H}_0 + \mathbf{H}_{hf} | + f_Q$

Electric field gradient is unaffected by the onset of SC or magnetism.

In(2a) : double broad peaks In(1),In(2b),Co : no dramatic change requirement from data

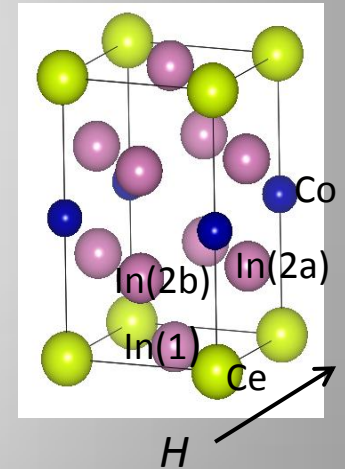


$$\mathbf{H}_{hf}(\mathbf{r}) = \sum_{i=1}^4 \frac{\mathbf{B}_i \cdot \mathbf{S}(\mathbf{r} + \mathbf{r}_i)}{\gamma \hbar} \text{ at In(1)}$$

$$\mathbf{H}_{hf}(\mathbf{r}) = \sum_{j=1}^2 \frac{\mathbf{B}_i \cdot \mathbf{S}(\mathbf{r} + \mathbf{r}_j)}{\gamma \hbar} \text{ at Co}$$

$$\mathbf{H}_{hf}(\mathbf{r}) = \sum_{k=1}^2 \frac{\mathbf{B}_i \cdot \mathbf{S}(\mathbf{r} + \mathbf{r}_k)}{\gamma \hbar} \text{ at In(2a)}$$

$$\mathbf{H}_{hf}(\mathbf{r}) = \sum_{l=1}^2 \frac{\mathbf{B}_i \cdot \mathbf{S}(\mathbf{r} + \mathbf{r}_l)}{\gamma \hbar} \text{ at In(2b)}$$



N. J. Curro *et al.*, J. Low. Temp. Phys. **158**, 635 (2010).

$\mathbf{H} // \mathbf{H}_{hf}$: blue line

$\mathbf{H}_0 \perp \mathbf{H}_{hf}$: red line

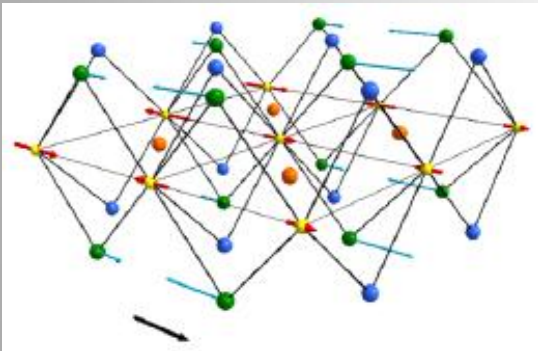
$$\mathbf{B}_i = \mathbf{B}_{iso} + \mathbf{B}_{dip} \quad \mathbf{Q} = \mathbf{Q}_0 + \mathbf{q}$$

Magnetic Structure

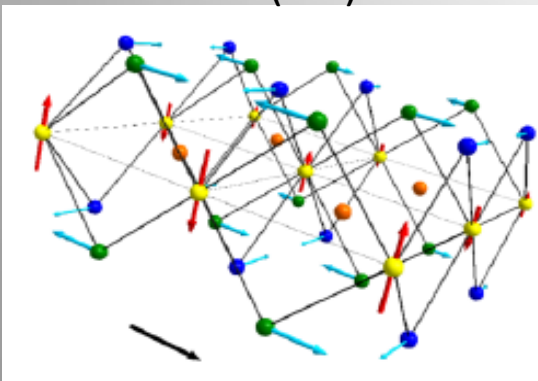
N. J. Curro *et al.*, J. Low. Temp. Phys. **158**, 635 (2010).

$$\mathbf{B}_{iso} = B_{iso} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \mathbf{B}_{dip} = \frac{B_{dip}}{r^2} \begin{pmatrix} 2x^2 - y^2 - z^2 & 3xy & 3xz \\ 3xy & -x^2 + 2y^2 - z^2 & 3yz \\ 3xz & 3yz & -x^2 - y^2 + 2z^2 \end{pmatrix}$$

Calculate \mathbf{H}_{hf} , assuming various \mathbf{S} , \mathbf{q} , \mathbf{B}_i under the condition of $H // [100]$



Case(1.1)



Case(2.4)

Case	\mathbf{q}	\mathbf{S}_0	\mathbb{B}	In(1)	In(2a)	In(2b)	Agreement?
(1.1)	[100]	[100]	iso	$\mathbf{H}_{hf} = 0$	$\mathbf{H}_{hf} // [100]$	$\mathbf{H}_{hf} = 0$	yes (a)
(1.2)	[100]	[100]	dip	$\mathbf{H}_{hf} // [010]$	$\mathbf{H}_{hf} // [001]$	$\mathbf{H}_{hf} = 0$	no
(1.3)	[100]	[001]	iso	$\mathbf{H}_{hf} = 0$	$\mathbf{H}_{hf} = 0$	$\mathbf{H}_{hf} // [001]$	no
(1.4)	[100]	[001]	dip	$\mathbf{H}_{hf} = 0$	$\mathbf{H}_{hf} // [100]$	$\mathbf{H}_{hf} // [010]$	yes (b)
(2.1)	[010]	[100]	iso	$\mathbf{H}_{hf} = 0$	$\mathbf{H}_{hf} = 0$	$\mathbf{H}_{hf} // [100]$	no
(2.2)	[010]	[100]	dip	$\mathbf{H}_{hf} // [010]$	$\mathbf{H}_{hf} // [001]$	$\mathbf{H}_{hf} \approx 0$	no
(2.3)	[010]	[001]	iso	$\mathbf{H}_{hf} = 0$	$\mathbf{H}_{hf} // [001]$	$\mathbf{H}_{hf} = 0$	no
(2.4)	[010]	[001]	dip	$\mathbf{H}_{hf} = 0$	$\mathbf{H}_{hf} // [100]$	$\mathbf{H}_{hf} // [010]$	yes (c)
(3.1)	[110]	[100]	iso	$\mathbf{H}_{hf} // [100]$	$\mathbf{H}_{hf} // [100]$	$\mathbf{H}_{hf} // [100]$	no
(3.2)	[110]	[100]	dip	$\mathbf{H}_{hf} // [001]$	$\mathbf{H}_{hf} // [100]$	$\mathbf{H}_{hf} // [010]$	no
(3.3)	[110]	[001]	iso	$\mathbf{H}_{hf} // [001]$	$\mathbf{H}_{hf} // [001]$	$\mathbf{H}_{hf} // [001]$	no
(3.4)	[110]	[001]	dip	$\mathbf{H}_{hf} // [010]$	$\mathbf{H}_{hf} // [100]$	$\mathbf{H}_{hf} // [010]$	yes (d)
(4.1)	$[1\bar{1}0]$	[100]	iso	$\mathbf{H}_{hf} // [100]$	$\mathbf{H}_{hf} // [100]$	$\mathbf{H}_{hf} // [100]$	no
(4.2)	$[1\bar{1}0]$	[100]	dip	$\mathbf{H}_{hf} // [001]$	$\mathbf{H}_{hf} // [100]$	$\mathbf{H}_{hf} // [010]$	no
(4.3)	$[1\bar{1}0]$	[001]	iso	$\mathbf{H}_{hf} // [001]$	$\mathbf{H}_{hf} // [001]$	$\mathbf{H}_{hf} // [001]$	no
(4.4)	$[1\bar{1}0]$	[001]	dip	$\mathbf{H}_{hf} // [010]$	$\mathbf{H}_{hf} // [100]$	$\mathbf{H}_{hf} // [010]$	yes (e)

G.L. free energy(mode coupling)

$$F = F_0 + \frac{a_M}{2} M^2 + \frac{b_M}{4} M^4 + \frac{a_\Psi}{2} \Psi^2 + \frac{b_\Psi}{4} \Psi^4 + \underline{C\Delta M\Psi}$$

M, Ψ, Δ : order parameter of SDW, π -pairing SC, d-wave SC ($a_M, b_M, a_\Psi, b_\Psi > 0$)

\leftrightarrow assuming that d-wave SC (not FFLO state) is necessary for SDW and π -pairing SC.

Minimize with respect to M, Ψ

$$\left[\begin{array}{l} \Psi = -\frac{C\Delta}{a_\Psi} M \\ M^2 = \frac{1}{b_M} \left[\frac{(C\Delta)^2}{a_\Psi} - a_M \right] \end{array} \right. \quad \begin{array}{l} \text{Approximation : } \Psi^3 \ll \Psi \quad (\text{considering CeCoIn}_5 \text{ is near the QCP}) \\ (C\Delta)^2 > a_\Psi a_M \end{array}$$

$\rightarrow M$ exists only in FFLO phase.

Wave vector q (Estimation of q and comparison with the data)

$$\mathbf{Q} = \mathbf{Q}_0 + \mathbf{q}$$

• Estimation : $q = \frac{2\mu_B H}{\hbar v_F} \cong 2.4 \times 10^8 \text{ m}^{-1} \frac{q}{Q_0} \cong 3.3 \times 10^{-2}$ • Data (Kenzelmann *et al.*) : $\frac{q}{Q_0} \cong 1.2 \times 10^{-2}$

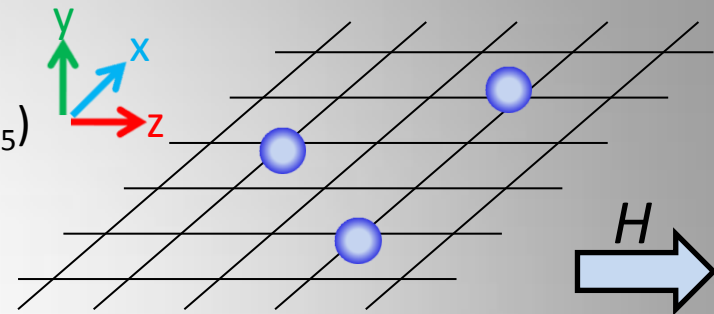
$\Delta Q = 2.1 \times 10^{-3}$: comparable to the error

• the possibility of H -dependence of V_F

\rightarrow The robustness of Q against H is understood.

model
$$H = -t \sum_{\langle i,j \rangle, \sigma} c_{i,\sigma}^\dagger c_{j,\sigma} + t' \sum_{\langle\langle i,j \rangle\rangle, \sigma} c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} + V \sum_{\langle i,j \rangle} n_i n_j + J \sum_{\langle i,j \rangle} S_i S_j - g_B H \sum_i S_i$$

- $\langle i,j \rangle$ ($\langle\langle i,j \rangle\rangle$) : summation over nearest (next nearest) site
- assuming square lattice (for quasi-two-dimensional CeCoIn₅)
- $H // z$



Approach 1 : BdG eq. (neglecting AFM fluctuation)

Approach 2 : FLEX approximation (considering AFM fluctuation)

Approach 1

Define mean field: $\langle S_i \rangle, \langle n_i \rangle, \Delta_{i,j}^{\sigma,\sigma'} = \langle c_{i\sigma} c_{j\sigma'} \rangle$

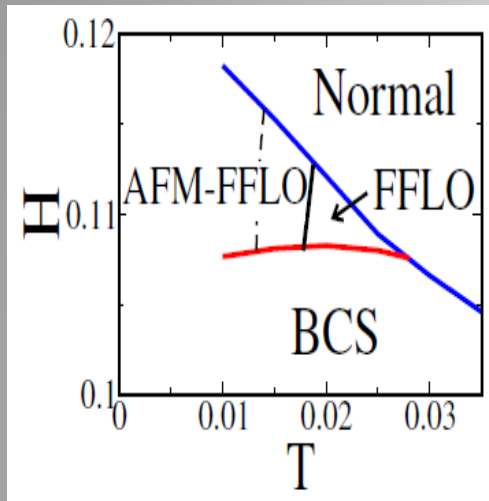
Staggered moment : $M_{AF}(i) = (-1)^{m+n} \langle S_i \rangle$

d-wave order parameter : $\Delta^d(i) = \Delta_{i,i+a}^{\uparrow\downarrow} + \Delta_{i,i-a}^{\uparrow\downarrow} - \Delta_{i,i+b}^{\uparrow\downarrow} - \Delta_{i,i-b}^{\uparrow\downarrow}$ (a, b : unit vectors along the z, x-axes)

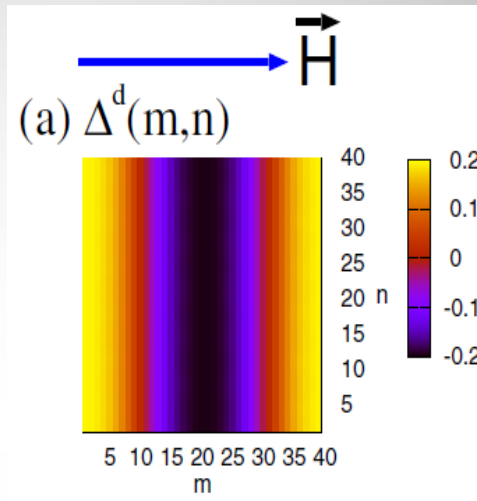
π -triplet order parameter : $d_{a,b}(i) \quad \Delta_{i,i\pm a}^{\sigma\sigma} = \pm (-1)^{m+n} \frac{1}{2} [-\sigma d_a^x(i) + i d_a^y(i)]$

$\Delta_{i,i\pm b}^{\sigma\sigma} = \pm (-1)^{m+n} \frac{1}{2} [-\sigma d_b^x(i) + i d_b^y(i)]$

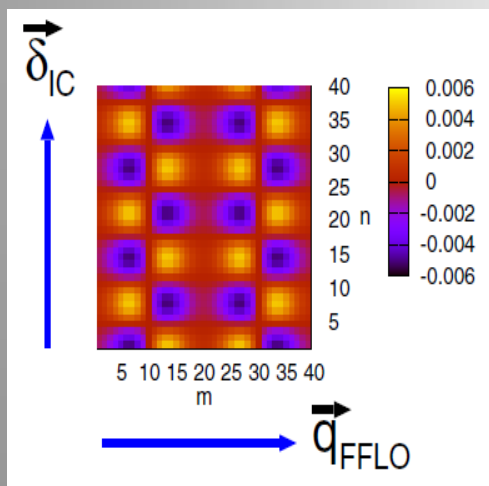
Then the thermodynamic average $\langle \dots \rangle$ (mean field Hamiltonian)
 \rightarrow self-consistent eq. for $\langle S_i \rangle, \langle n_i \rangle$, and $\Delta_{i,\delta}^{\sigma,\sigma'}$ for each i : BdG eq.



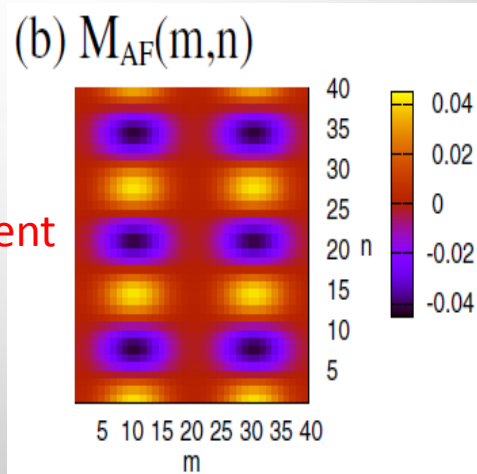
- AFM only in FFLO
- **Andreev bound state**
- Normal state to SC is first order in high field. $\uparrow U$ and J are required.
- $T_c(H=0) = 0.096$
- dashed line : π -triplet is neglected \rightarrow important for AFM



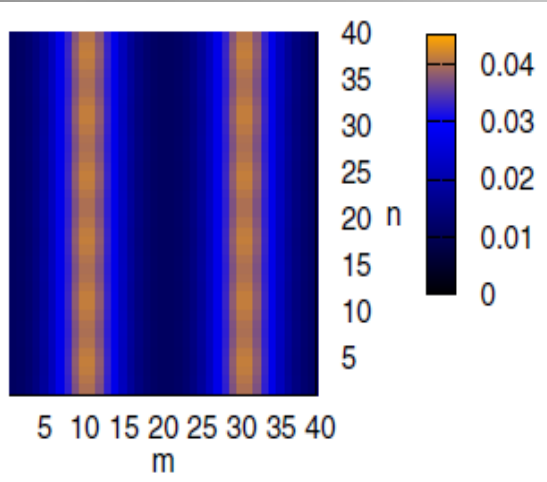
- Modulated order parameter
- large DOS around the nodes



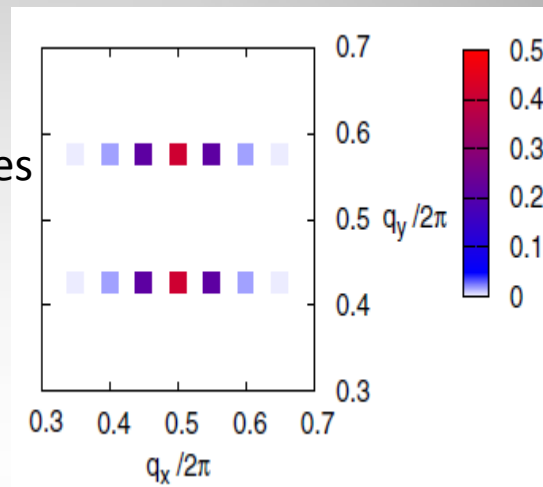
- $\delta_{IC} \perp q_{FFLO} // H$
- $|\delta_{IC}|$: independent of nodal planes
- **consistent with experiment**



- AFM order occurs with nucleation of the planar nodes
- localized around the nodes
- $q_{FFLO} // H$ (assumption) $\rightarrow \delta \perp H, M \perp H$



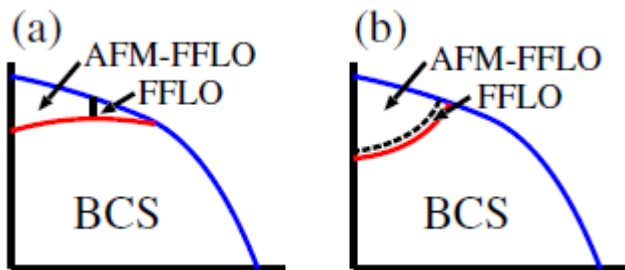
- weak modulation along the nodal lines
- IC-AFM order



Fourier transformation of $\langle S_i \rangle$

- Peaks at $Q = Q_0 \pm \delta_{IC}$
- ✂ Remark
- Satellite peaks

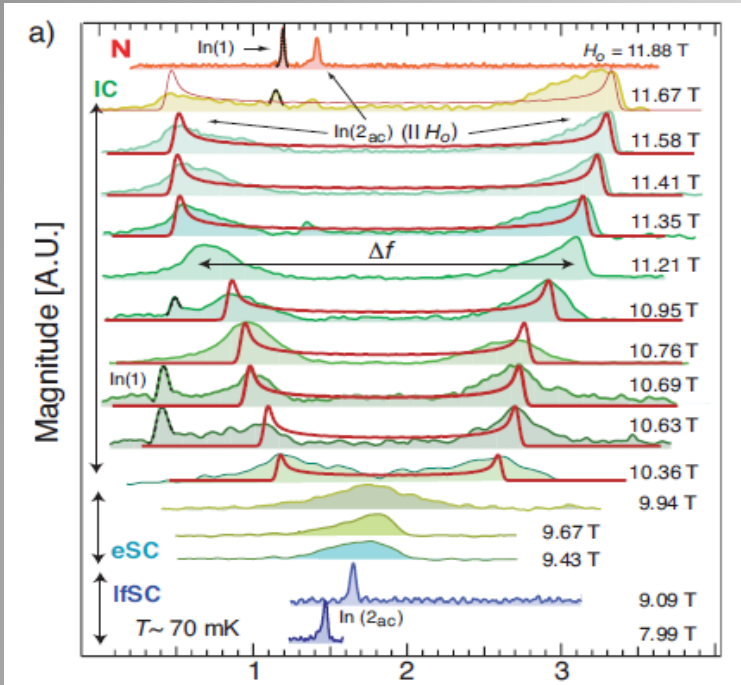
Approach 2 $\langle S_i \rangle$



- AFM order : from three to quasi-2-dimensions near the BCS-FFLO transition line
- enhanced spin fluctuation
- Pure FFLO state without AFM order**

Findings : Inhomogeneous FFLO state → Andreev bound state → stabilized AFM (π -triplet SC)

New phase near the AFM-FFLO state, where strong spin fluctuation exists



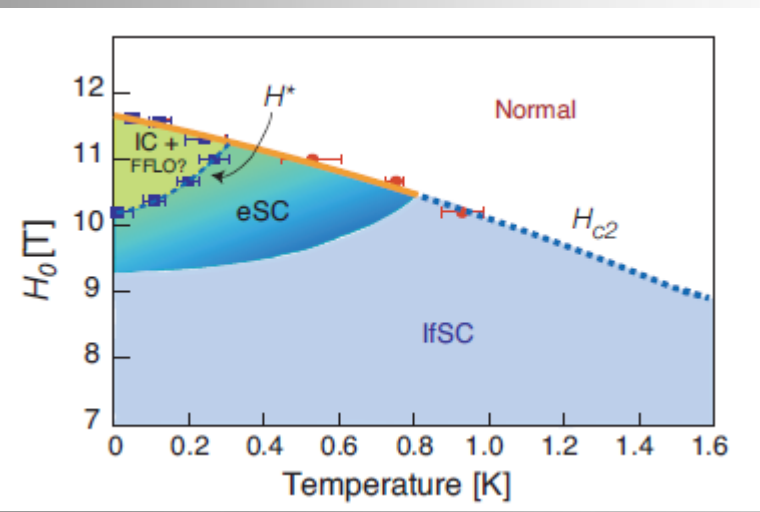
In(1), In(2_{ac}), In(2_{bc}) site $H // [100]$

- In(2_{ac}) : broadening, double peaks
- In(1), In(2_{bc}) : not broadening

$$\mu_i // \mathbf{c} \perp \mathbf{H} \quad \mu_0 = 0.15 \mu_B$$

$$\delta // \mathbf{b} \perp \mathbf{H} \quad \delta = 0.085 \quad \rightarrow \text{consistent with data}$$

Direction of δ : H -independent

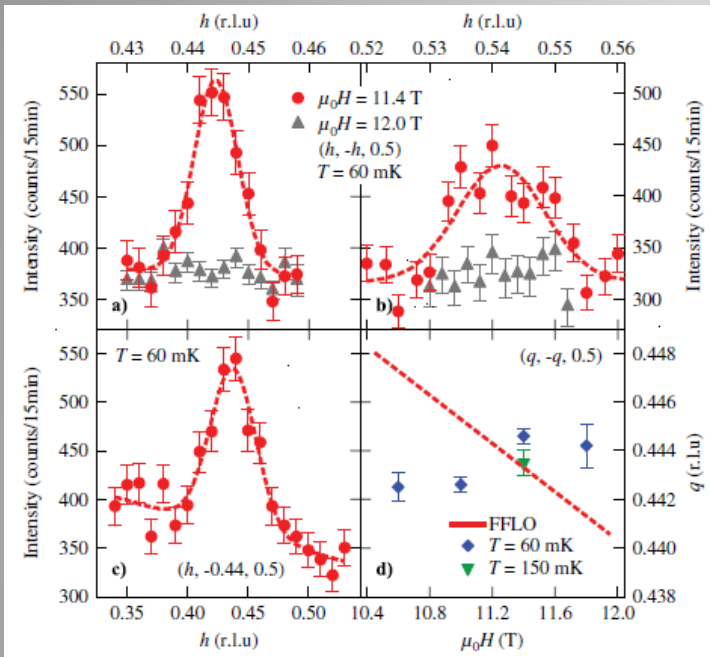


- In(2ac) : weak signal, disappearance of the double peak
- New phase under the IC+FFLO phase
- Very strong AFM fluctuation

Neutron diffraction measurement

M. Kenzelmann *et al.*, Phys. Rev. Lett. **104**, 127001 (2010).

$H // [100] (h, h, 0.5)$



- correlation length $> 25 \text{ nm} > 10 \text{ nm}$ (vortex core scale)
- magnetic structure

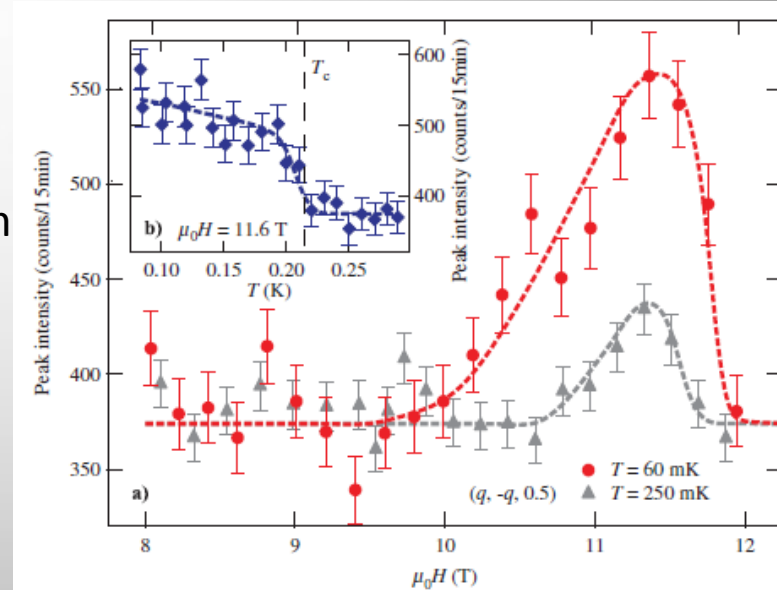
$$\mathbf{Q} = (q, -q, \frac{1}{2}) \quad q = 0.45 \quad \mathbf{S} \perp \mathbf{H} // [100] \quad \mu_0 = 0.16 \mu_B$$

q : H -independent \rightarrow inconsistent with FFLO

Direction of \mathbf{Q} : parallel to the direction of node?

$$d_{x^2-y^2}$$

[11/] electron nesting \rightarrow SDW



SDW originates from local magnetism in vortex cores.

High DOS of low-energy excitation that induces magnetism

\rightarrow Enhanced vortex magnetism

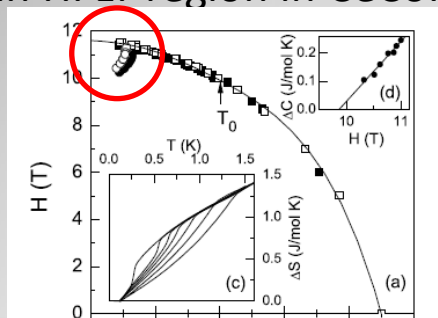
Vortexes ($\sim 13\text{-}15 \text{ nm}$) almost overlap in high field.

\rightarrow long-range SDW

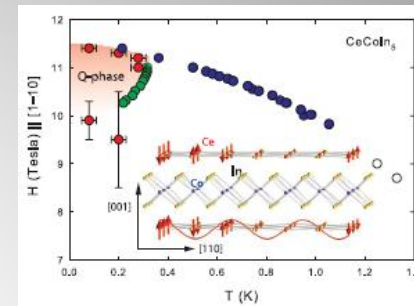
SC and SDW vanish at the same time, H_{c2} : consistent

Summary

- New phase was found in HFLT region in CeCoIn_5 , and AFM coexists with SC in that region.

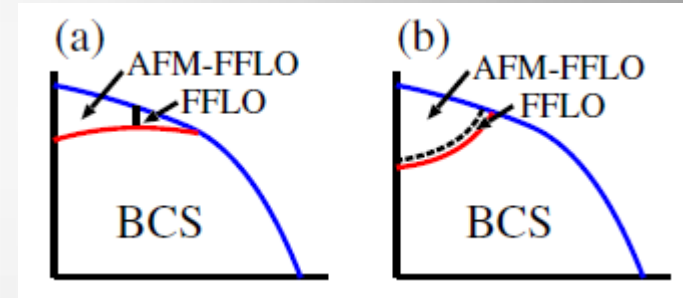


A. Bianchi *et al.*, Phys. Rev. Lett. **91**, 187004 (2003).



M. Kenzelmann *et al.*, Science **321**, 1652 (2008).

- Theories predict that FFLO state (inhomogeneous) and AFM can coexist and moreover, the phase is FFLO state-driven.



Y. Yanase *et al.*, J. Phys. Soc. Jpn. **78**, 114715 (2009).

- Neutron scattering experiment indicates that the phase is magnetic-driven and the data is inconsistent with theories above.