Penetration depth measurements in URu_2Si_2

Geometric factors and fitting by a two-band model with chiral d-wave gap symmetry

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Summer Seminar, Matsuda Lab.
"Soutenance de stage d'option scientifique", Ecole Polytechnique

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Contents

Tunnel diode oscillator

First proposed by van Degrift, 1981 for high precision measurements of resonance frequency, sample inserted into the primary coil.

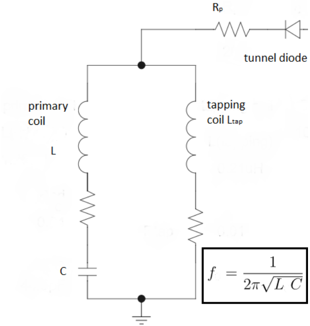

Figure: oscillating circuit, part of the low-T electronics

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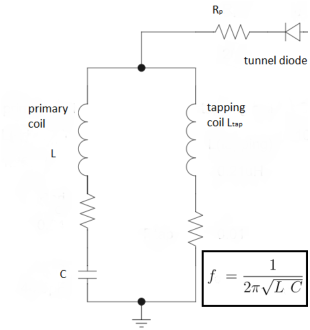

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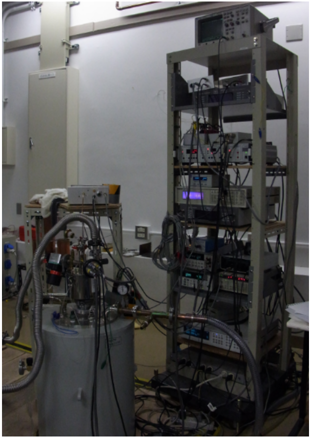

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How can we relate the resonance frequency and λ?

- start with magnetic energy of coil (SI)

$$U = \frac{1}{2} L I^2 = \frac{1}{2} \int \vec{B} \cdot \vec{H} d^3 r$$

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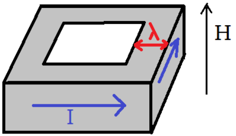

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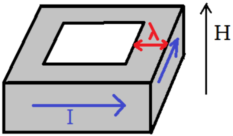

- for the empty coil: $\frac{1}{2}LI^2 = \frac{B_0^2 V_c}{2\mu_0}$, eliminate I
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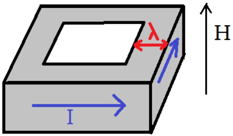

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Resulting frequency change with temperature

$$f(T) - f(T_{min}) = -\frac{f_0}{2V_c}\int_{V_s}\frac{M(\lambda(T),H_0) - M(\lambda(T_{min}),H_0)}{H_0}d^3r$$

$M(\lambda)$ following Chia, 2004, Prozorov et al., 2000

1. take into account the the demagnetizing effect

$$M = \frac{\chi}{1 + N\chi} H_{applied}$$

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Demagnetizing factor N, (see [Brandt, 2001a])

Strictly defined only for ellipsoids (homogeneous M)

$$\begin{aligned} H_{intern} &= H_{applied} - NM(H_{intern}; N = 0) \\ M(H_{intern}; N = 0) &= M(H_{applied}; N) \\ M(H_{intern}; N = 0) &= \chi H_{intern} \end{aligned}$$

An effective demagnetizing factor can also be defined (via average magnetization) for other shapes.
See Refs. [Osborn, 1945] for the general ellipsoid, [Chen et al., 1991] for cylinders, or [Pardo et al., 2004] for rectangular prisms.

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$\chi = M/H = \left[1 - \frac{\lambda}{w} \tanh\left(\frac{w}{\lambda}\right)\right]$

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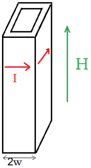

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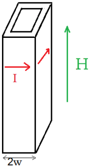

For a finite sample an effective dimension can be introduced [Prozorov et al., 2000]

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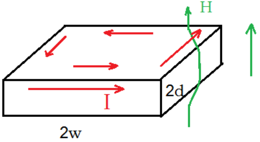

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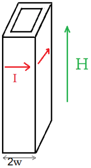

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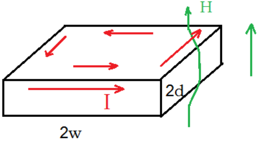

3. We linearize:

$$M = \frac{\chi}{1 - N} H$$

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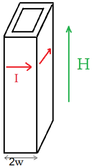

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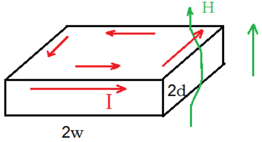

resulting magnetization

$$M = -\frac{H}{1 - N}\left[1 - \frac{\lambda}{R_{3D}}\right]$$

Conclusion for “Prozorov-G-factor”

Summary until now

$$f(T) - f(T_{min}) = -\frac{f_0}{2V_c} \int_{V_s} \frac{M(\lambda(T), B_0) - M(\lambda(T_{min}), B_0)}{H_0} d^3 r$$

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By putting everything together

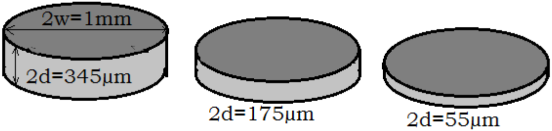
$$G = \frac{2(1-N)R_{3D}}{f_0}\frac{V_c}{V_s} \text{ so that } \Delta\lambda = G\Delta f$$

- demagnetizing factor $0 < N < 1$
- effective dimension $0.2w < R_{3D} < 0.5w$
- filling factor V_s/V_c

How to test this approach?

Sample dependance

$$G \propto \frac{(1-N)R_{3D}}{V_s}$$



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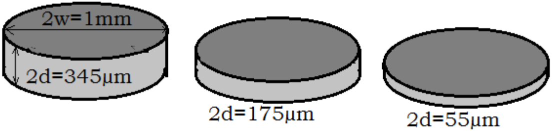

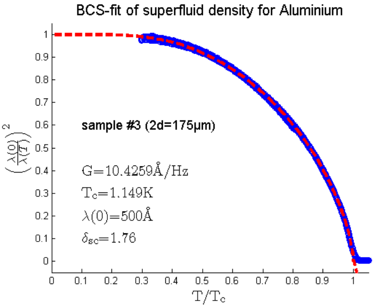

$$\frac{\lambda(T)}{\lambda(0)} = \left[\frac{\Delta(T)}{\Delta(0)} \tanh \frac{\Delta(T)}{\Delta(0)} \frac{\delta_{sc} T_c}{2T}\right]^{-1/3}$$

$$\frac{\Delta(T)}{\Delta(0)} = \tanh\left(\frac{\pi}{\delta_{sc}}\sqrt{a\left(\frac{T_c}{T}-1\right)}\right)$$

where: $\Delta(0) = \delta_{sc} k_B T_c$

[Tinkham, 1996, Prozorov and Giannetta, 2006]

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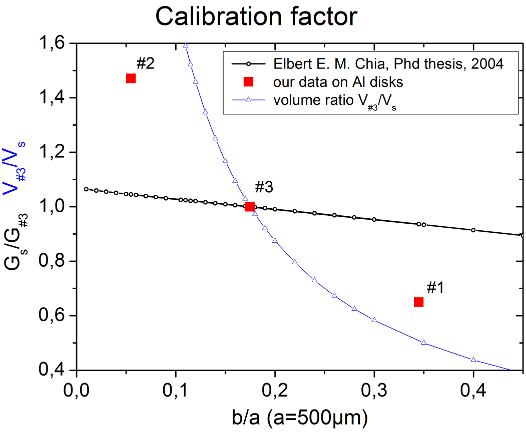

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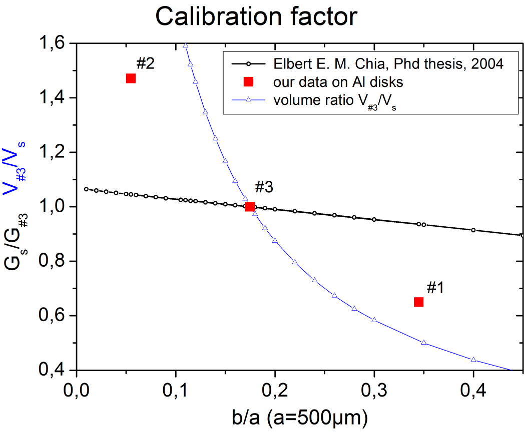

Something is wrong...

Demagnetizing effects exist but are greatly overestimated!

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$$-\lambda^2\mu_0\vec{j} = \vec{A} = \vec{A}_j + \vec{A}_a \quad (1)$$

$$\mu_0 j = -\nabla^2 A_j \quad (2)$$

- A_a comes from the applied field H_a. $A_a = -\frac{r}{2}\mu_0 H_a$ for the disk case.
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3. Solve this equation numerically on a grid.

Compute on a grid...

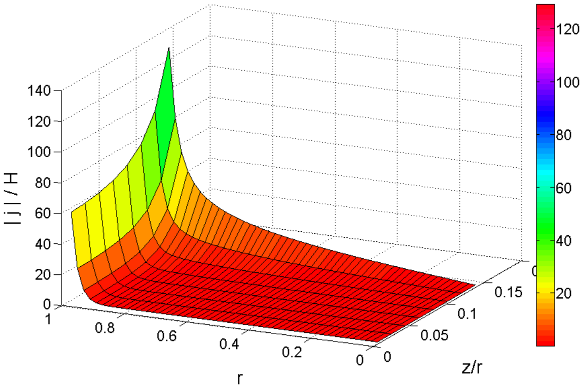

Figure: Example of a simulation for a disk with radius $r = 1$, half-height $d = 0.15$, $\lambda = 0.1$ on a small grid with 480 equally spaced gridpoints

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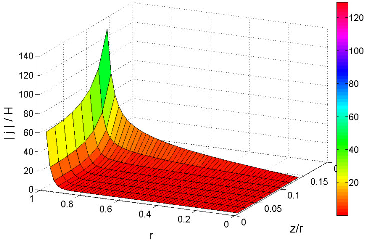

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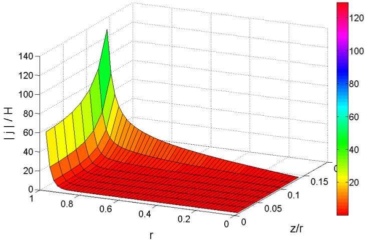

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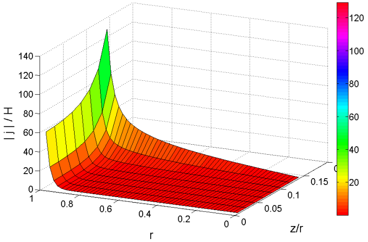

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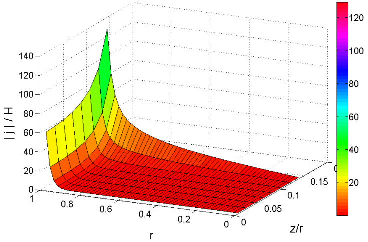

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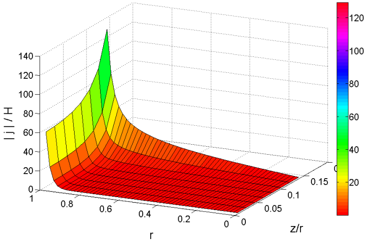

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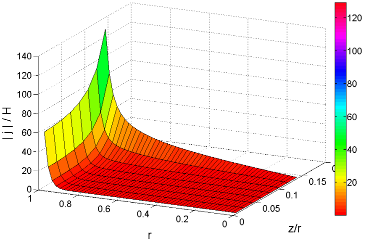

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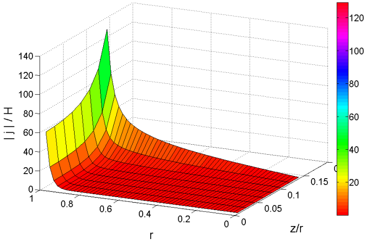

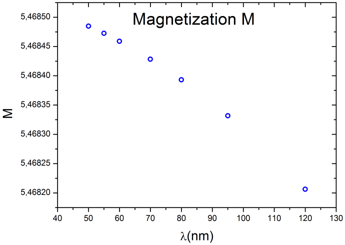

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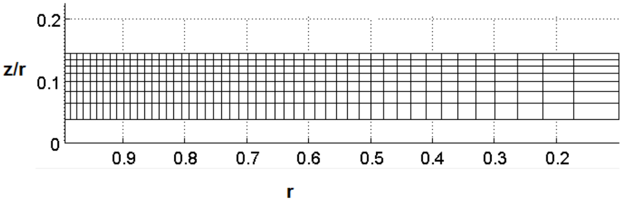

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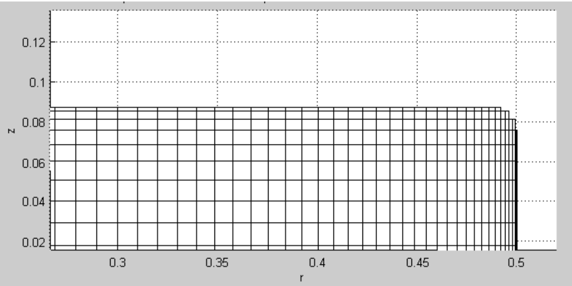

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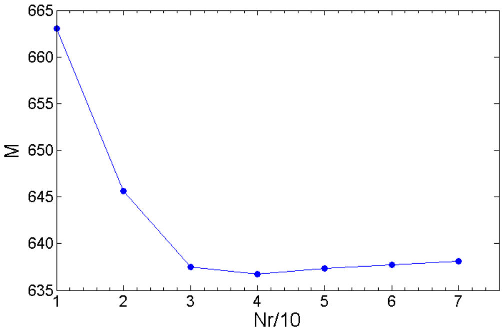

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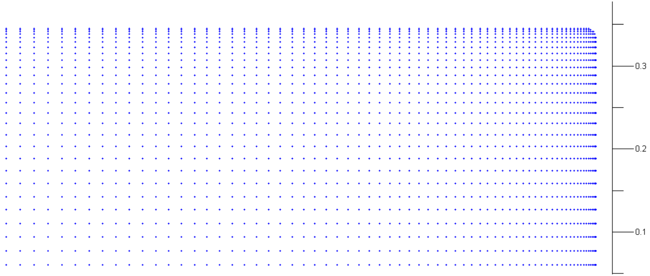

Figure: Example of the final grid with $\sim$ 2000 gridpoints

Some problems and some solutions

- Use a grid that gets finer at the borders (unequal weights)
- Remove corner points to model realistic corners
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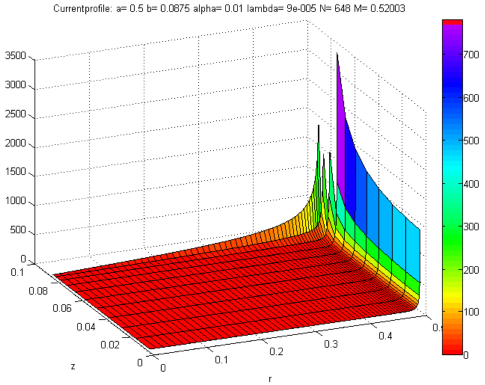

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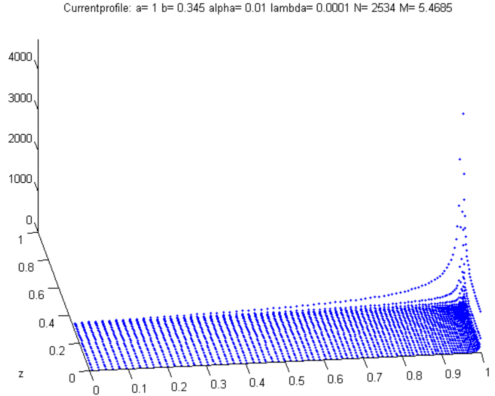

Results of computational approach for Al-disks

$$f(T) - f(T_{min}) = -\frac{f_0}{2V_c}\left(\int_{V_s} M(\lambda(T), H_0)/H_0 d^3r - \int_{V_s} M(\lambda(T_{min}), H_0$$

$$f(T) - f(T_{min}) \approx \frac{f_0}{2V_c} V_s \frac{dM}{d\lambda}(\lambda(T_{min}) - \lambda(T))/H_0$$

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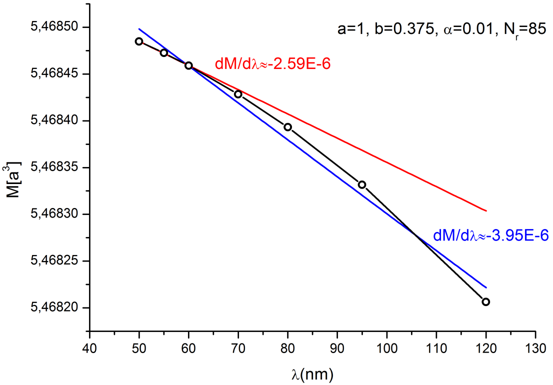

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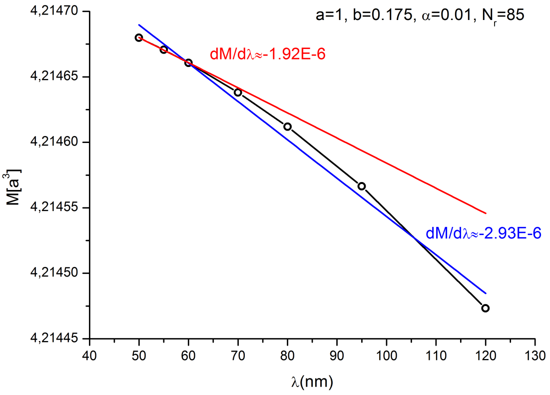

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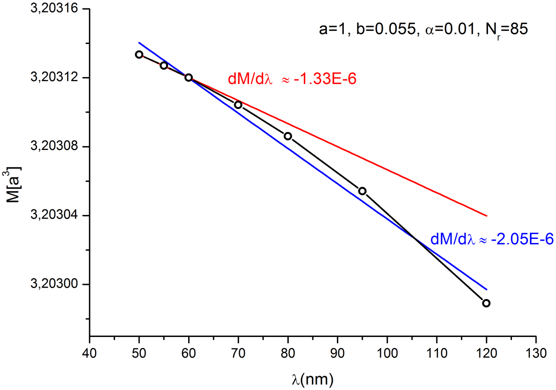
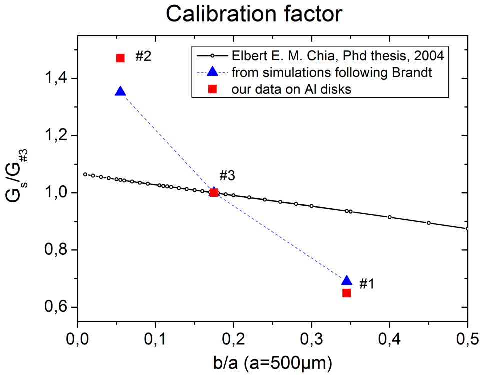


Results of computational approach for Al-disks

Check obtained relative G-factors



- Reasonable agreement!
- However, absolute values cannot be reproduced.

Trying on the URu_2Si_2-samples

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#E2_A: $G = 250$ Å/Hz found, $G \approx 200$ Å/Hz probably correct!
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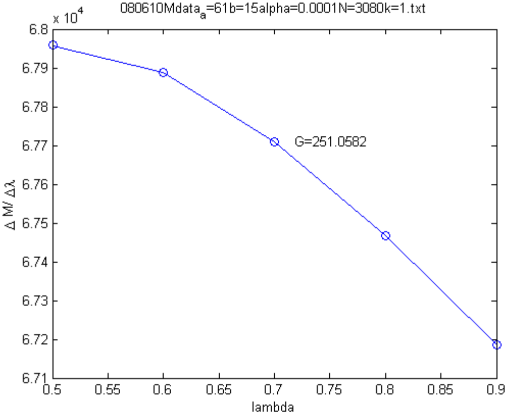

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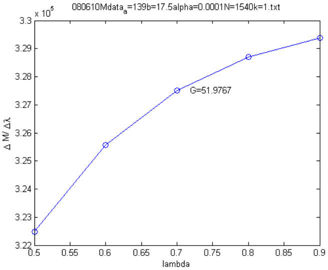

$\frac{dM}{d\lambda}$ for λ appropriate for URu_2Si_2:
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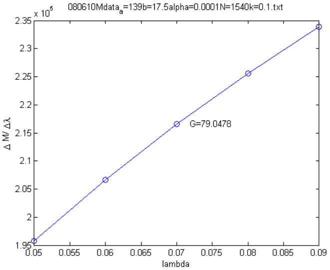

$\frac{dM}{d\lambda}$ for λ appropriate for Aluminium:
$G_{Al} = 79$!

Adapted final solution

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Cut an Al-sample of the same size/shape as our best URu_2Si_2-sample (#E2_B)

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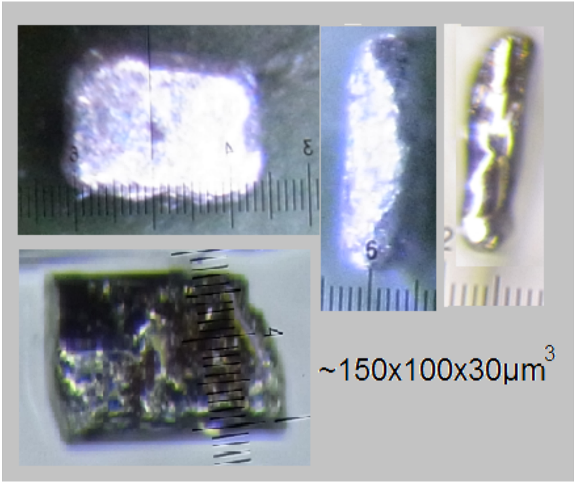

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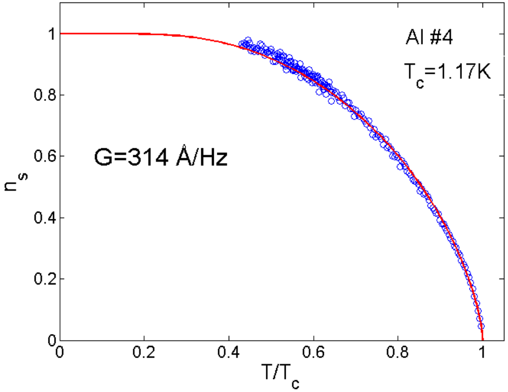

Contents

URu_2Si_2

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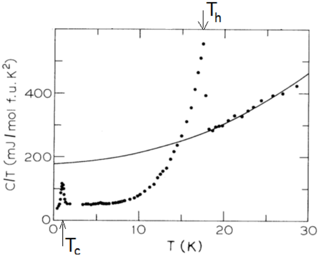

Figure: taken from Ref. [Palstra et al., 1985]

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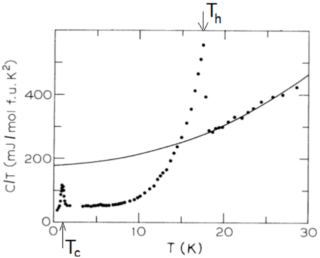

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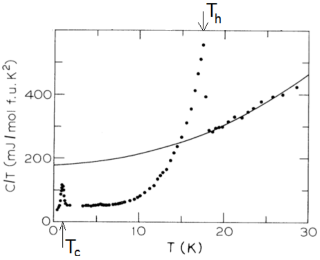

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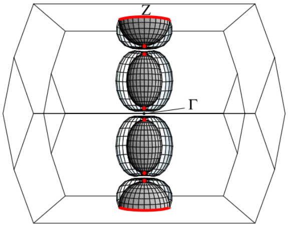
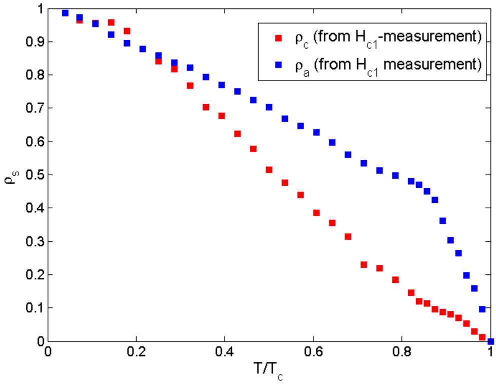
- Thermal conductivity measurements by Kasahara et al.
- Specific heat measurements by Yano et al.

$\Delta_k \propto k_z(k_x + ik_y)$ (chiral d-wave)



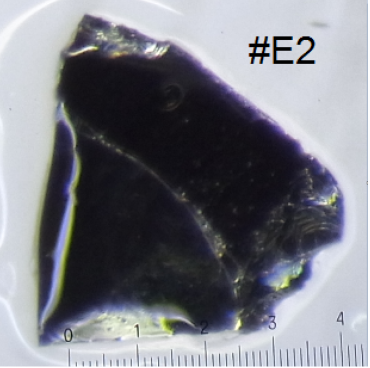
H_{c1} and mysterious kink

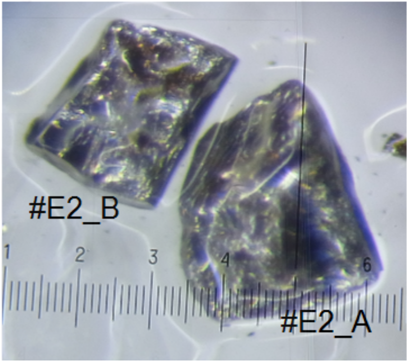
See [Okazaki et al., 2009]
Superfluid density extracted from H_{c1}



Our samples

- ultraclean ($RRR \sim 700$)
- Cut one larger cristal into two.





How to get $\Delta\lambda_{ab}$ and $\Delta\lambda_c$

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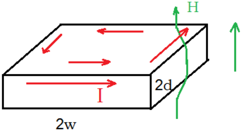

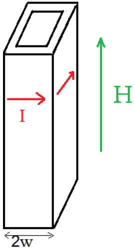

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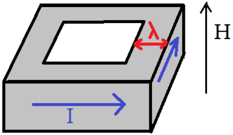
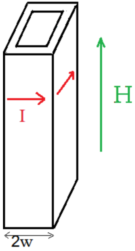
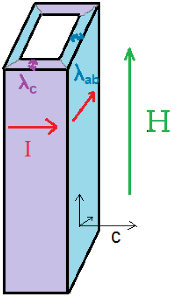


$$\Delta\lambda_{ab} = G\Delta f_{H||c}$$

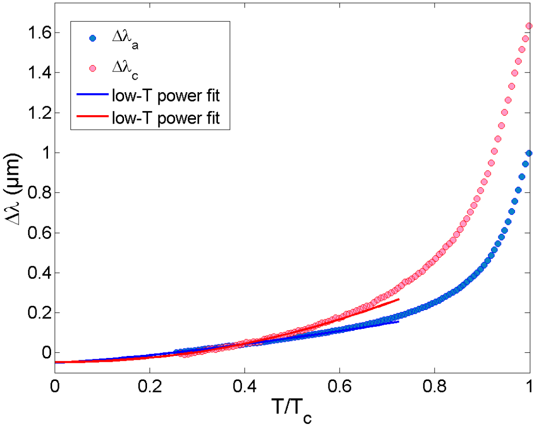




$$\Delta f_{H||a} = \frac{f_0 V_s}{2V_c}\left(\frac{\Delta\lambda_{ab}}{d} + \frac{\Delta\lambda_c}{w}\right)$$

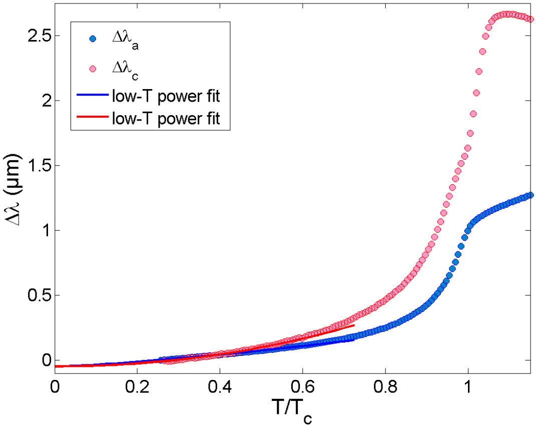
Result

We can find the change in penetration depth:



Result

We can find the change in penetration depth:
But strangely we get different critical temperatures.



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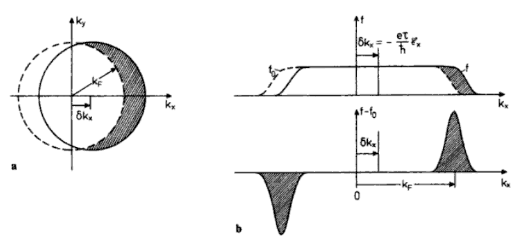

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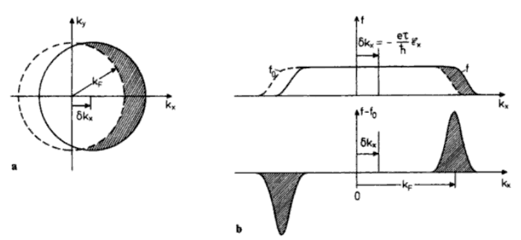

$$\vec{j} \propto \int d^3k\vec{v}_k\left(f(\epsilon_k - \delta\epsilon_k) - f(\epsilon_k + \delta\epsilon_k)\right)$$

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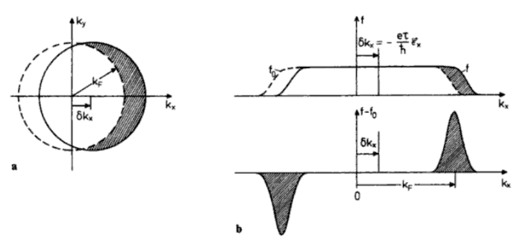

$$\vec{j} \propto \int d^3k \vec{v}_k \underbrace{\left(f(\epsilon_k - \delta\epsilon_k) - f(\epsilon_k + \delta\epsilon_k)\right)}_{2\left(-\frac{\partial f}{\partial \epsilon_k}\right)\delta\epsilon_k}$$

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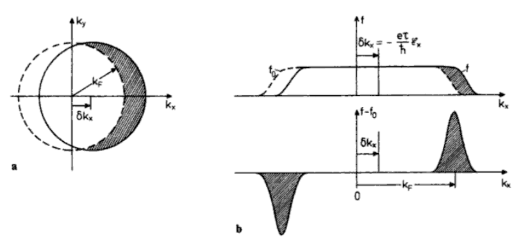

$$\vec{j} \propto \int d^3k\left(-\frac{\partial f}{\partial\epsilon_k}\right)\tau_k(\vec{v}_k\vec{v}_k)\cdot\vec{E} \Rightarrow \sigma_{im} \propto \oint dS_F \frac{v_{Fi}v_{Fm}}{v_F}\tau_F$$

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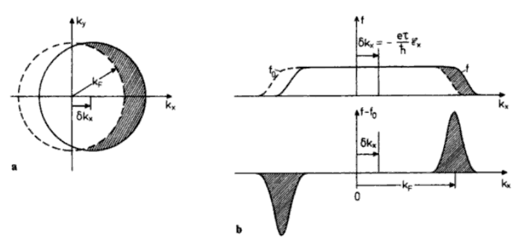

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For (quasi)free electrons and isotropic τ we find Drude: $\sigma = \frac{ne^2\tau}{m^*}$

Let's do the same for the supercurrent

[Chandrasekhar and Einzel, 1993]

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Normal conductivity

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London equations:

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n_k is the single particle occupation of the state k

$f(E_k)$ is its occupation by quasiparticles. $E_k = \sqrt{\epsilon_k^2 + \Delta_k^2}$

Only superconducting electrons contribute to j_s.

How do we get the superfluid-density now?

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$$\vec{j}_s \propto -\int d^3k \left[-\left(\frac{\partial n_k}{\partial \epsilon_k} - \frac{\partial f(E_k)}{\partial E_k} \right) \right] (\vec{v}_k \vec{v}_k) \cdot \vec{A}$$

We use:

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- Second (paramagnetic) term: $\oint dS_F \frac{\vec{v}_F \vec{v}_F}{v_F} 2 \int \frac{d\epsilon_k}{dE_k} dE_k \frac{\partial f(E_k)}{\partial E_k}$

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$$d\epsilon_k = 2 \frac{d\epsilon_k}{dE_k} dE_k$$

- First (diamagnetic) term: $\oint dS_F \frac{\vec{v}_F \vec{v}_F}{v_F}$
- Second (paramagnetic) term: $\oint dS_F \frac{\vec{v}_F \vec{v}_F}{v_F} 2 \int \frac{d\epsilon_k}{dE_k} dE_k \frac{\partial f(E_k)}{\partial E_k}$

Final result $\mu_0 \vec{j} = -\lambda^{-2} \vec{A}$ [Prozorov and Giannetta, 2006]

$$\lambda_{ij}^{-2}(T) = \frac{\mu_0 e^2}{4\pi^3 \hbar} \oint_{FS} dS_k \left[\frac{v_F^i v_F^j}{|v_F|} \left(1 + 2 \int_{\Delta(k)}^{\infty} dE_k \frac{\partial f(E_k)}{\partial E_k} \frac{N(E_k)}{N(0)} \right) \right]$$

(Only!) for quasi-free electrons and “simple” geometry we find

$$\lambda(0)^{-2} = \frac{\mu_0 n e^2}{m^*}$$

How do we get the superfluid-density now?

$$\vec{j}_s \propto -\int d^3k \left[-\left(\frac{\partial n_k}{\partial \epsilon_k} - \frac{\partial f(E_k)}{\partial E_k}\right)\right](\vec{v}_k\vec{v}_k)\cdot\vec{A}$$

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We can now compute the superfluid density

$$\rho_{ii} = \left(\frac{\lambda_{ii}(0)}{\lambda_{ii}(T)}\right)^{-2}$$

What does it look like in our case?

$$\lambda_{ij}^{-2}(T) = \frac{\mu_0 e^2}{4\pi^3 \hbar} \oint_{FS} dS_k \left[\frac{v_F^i v_F^j}{|v_F|} \left(1 + 2 \int_{\Delta(k)}^{\infty} dE_k \frac{\partial f(E_k)}{\partial E_k} \frac{N(E_k)}{N(0)} \right) \right]$$

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a little scary.

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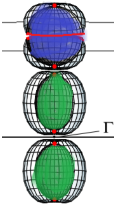

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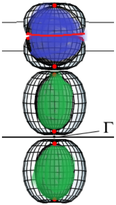

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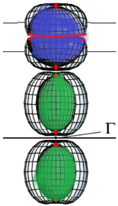

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$$H_{c2} = \frac{\Phi_0}{2\pi\xi^2}$$
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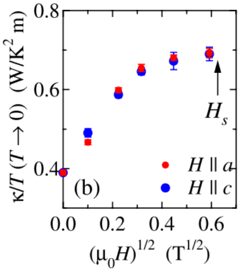

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- use the "virtual upper critical field" (see [Kasahara et al., 2007]) for the light hole band
- Correct for the size of gaps

$$\frac{v^a_{F,e}}{v^a_{F,h}} = \sqrt{\frac{H_s}{H_{c2}^{H||a}}} \frac{\Delta_{0,e}}{\Delta_{0,h}}$$

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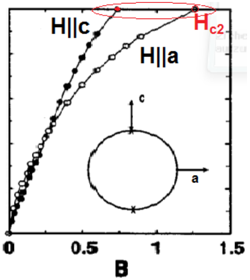

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- Correct for point nodes in the electron FS (see [Miranović et al., 2001])

$$\frac{v^c_{F,e}}{v^a_{F,e}} = \frac{H_{c2}^{H||a}}{H_{c2}^{H||c}} \cdot 1.66$$

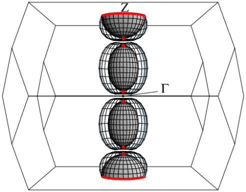
Let's apply this to data

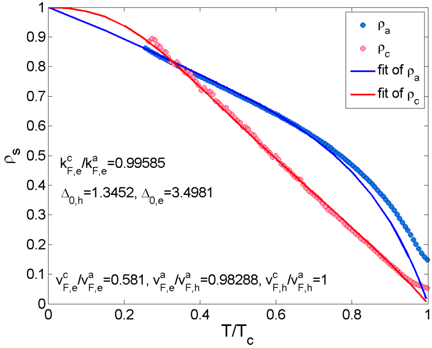
Parameters needed:

- Shape of FS: $\frac{k^h_{F,c}}{k^h_{F,a}}$, $\frac{k^e_{F,c}}{k^e_{F,a}}$
- Anisotropy of v_F for each FS
- Ratio of v_F of the 2 bands

Fitting parameters:

- Gapvalues Δ_0 (α-model)
- anisotropy of e-FS





And a different gap structure?

Try to find the nodel structure in a 2-band model with equal volume of the FS's
vspace10cm

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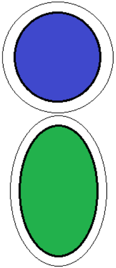
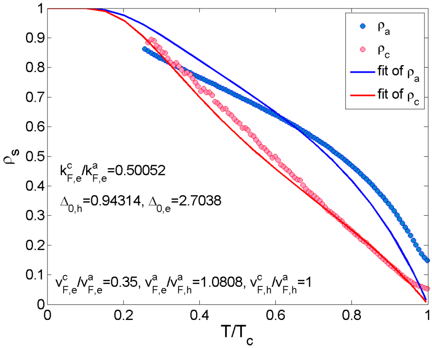
Both isotropic s-wave

$$\Delta_h = \Delta_h(T)$$
$$\Delta_e = \Delta_e(T)$$

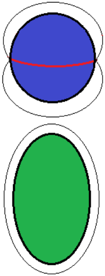


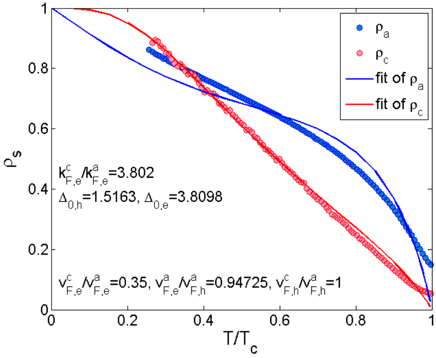


And a different gap structure?

Try to find the nodel structure in a 2-band model with equal volume of the FS's
hole-FS: only line node, electron-FS: fully gapped

$$\Delta_h = \Delta_h(T)\cos(\theta)$$
$$\Delta_e = \Delta_e(T)$$





And a different gap structure?

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hole-FS: line node, electron-FS: point nodes

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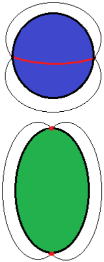
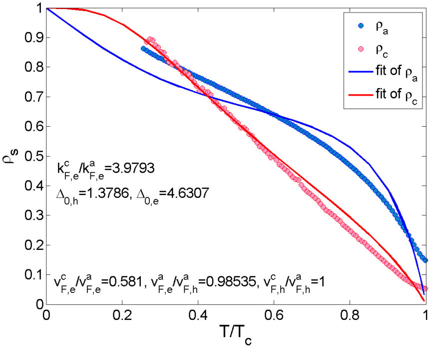


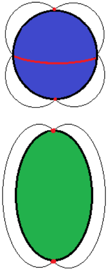


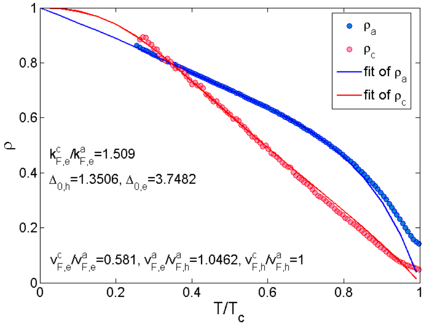
And a different gap structure?

Try to find the nodel structure in a 2-band model with equal volume of the FS's

hole-FS: line node and point nodes, electron FS: point nodes

$$\Delta_h = \Delta_h(T) 2\cos(\theta)\sin(\theta)$$
$$\Delta_e = \Delta_e(T)\sin(\theta)$$





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Both line node and point nodes

$$\Delta_h = \Delta_h(T) 2\sin(\theta)\cos(\theta)$$
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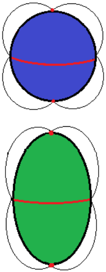

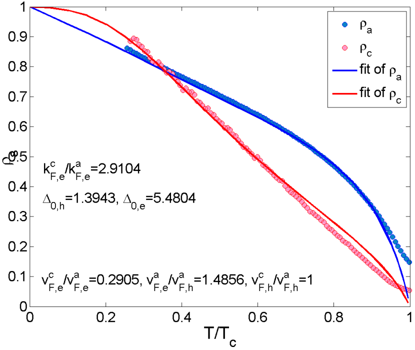

What about Fermi-liquid corrections?

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Predicted behaviour: [Varma et al., 1986]

$$\lambda^{-2}(T) = \frac{\mu_0 n e^2}{m_d}\left[1 - \frac{(1 + F_1^s/3)Y(T)}{1 + (F_1^s/3)Y(T)}\right]$$

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- m_d: dynamic mass, for example from cyclotron resonance
- F_1^s Landau coefficient
- $Y(T)$ is the Yosida function, $N(0)^{-1}\sum_k dn/dE_k$

What about Fermi-liquid corrections?

Limiting behaviour: [Varma et al., 1986]

$$\lambda^{-2}(T) = \frac{m}{m_d}\lambda_{BCS}^{-2}(T), \quad T \to 0$$

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We estimate $\lambda(0)$ using cyclotron resonance mass.

$$\lambda(0) = \left(\frac{\mu_0 n e^2}{m_d}\right)^{-1/2}$$

We get $\lambda_a(0) = 0.66\mu m$ and $\lambda_c(0) = 0.5\mu m$, smaller than $\lambda \approx 1\mu m$ from μSR and $\lambda_a(0) > \lambda_c(0)$ in contrast to H_{c1}-measurements.

May Fermi-liquid corrections play a role?

$$\lambda^{-2}(T) = \frac{\mu_0 n e^2}{m_d}\left[1 - \frac{(1+F_1^s/3)Y(T)}{1+(F_1^s/3)Y(T)}\right]$$

Generalize this to include bandstructure (*quite* a long formula)

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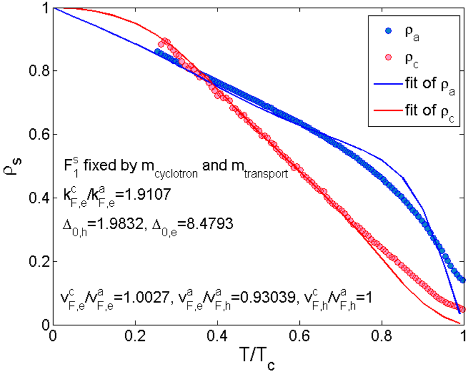
The F_1^s fixed by mass ratios m^*/m_d, all other parameters free



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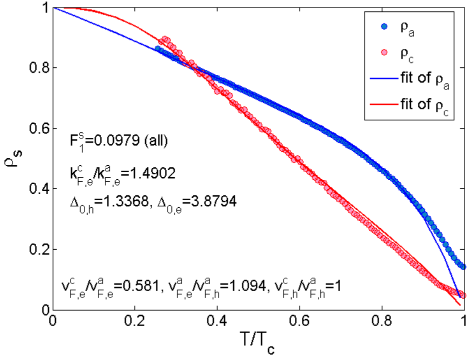
Fit F_1^s starting from 0



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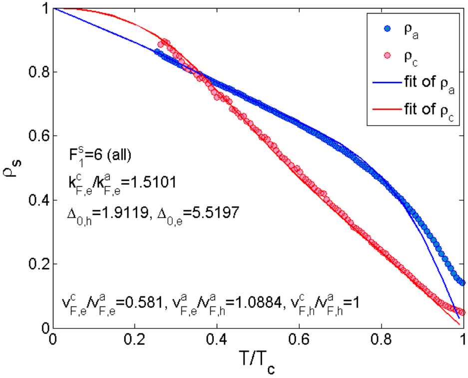
Set all $F_1^s = 6$



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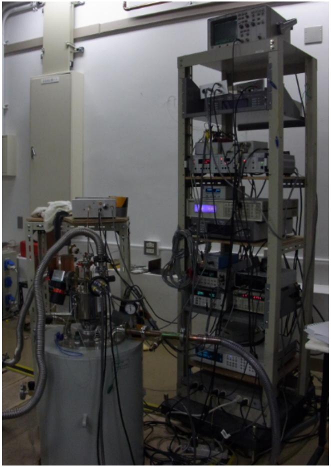

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- The temperature dependent penetration depth can be measured accurately with TDO technique
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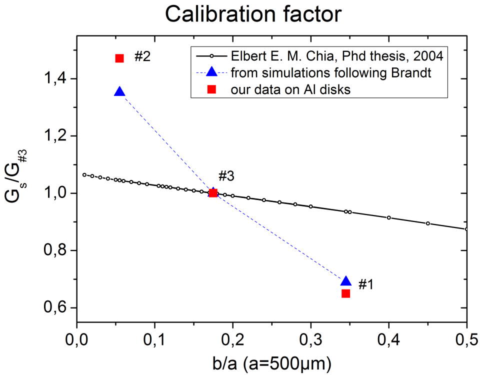

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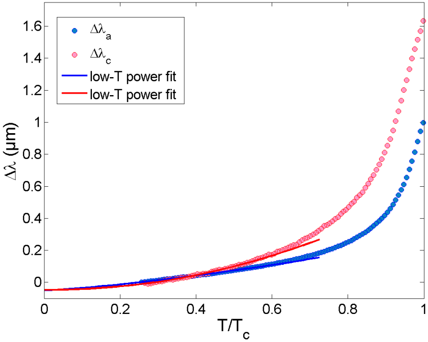

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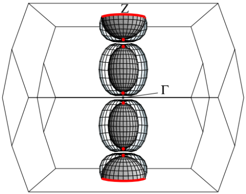

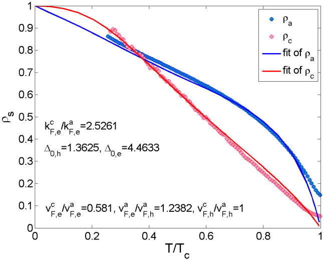

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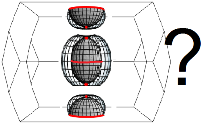
- FL-corrections possibly play a role in the T-dependance of λ, difficult to be sure
- Unknown Fermi-surface implies many difficulties.



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